Most structural systems constructed today are made from reinforced, prestressed, or composite concrete having a wide range of characteristics and strengths. Structural concrete, whether normal weight or lightweight, is designed to have a compressive strength in excess of 3000 psi (20 MPa) in concrete structures. When the strength exceeds 6000 psi (42 MPa) such structures are defined today as high-strength concrete structures. Concrete mixtures designed to produce 6000 to 12,000 psi in compressive strength are easily obtainable today when silica fume or other pozzolans replace a portion of the cement content, resulting in lower water/cement (w/c) and water/cementitious materials (w/cm) ratios. Concretes...
having cylinder compressive strengths of about 20,000 psi (140 MPa) have been used in several buildings
in the United States. These high-strength characteristics merit qualifying such concrete as super-high-
strength concrete at this time.

### 36.1 Material Characteristics

#### 36.1.1 Modulus of Concrete

The ACI 318 Code (ACI Committee 318, 2008; Nawy, 2002, 2008) stipulates that the concrete modulus
of elasticity \( E_c \) should be evaluated from:

\[
E_c (\text{psi}) = 33w^{1.5} \sqrt{f'_c} \\
E_c (\text{MPa}) = 0.043w^{1.5} \sqrt{f'_c}
\]

The expressions in Equation 36.1 are applicable to strengths up to 6000 psi (42 MPa). Available research
to date for concrete compressive strength up to 12,000 psi (83 MPa) gives the following expressions (ACI

\[
E_c (\text{psi}) = \left(40,000 \sqrt{f'_c} + 10^4\right) \left(\frac{w_c}{145}\right)^{1.5} \\
E_c (\text{MPa}) = \left(3.32 \sqrt{f'_c} + 6895\right) \left(\frac{w_c}{2320}\right)^{1.5}
\]

In Equation 36.1a and Equation 36.2a, \( f'_c \) is in units of pounds per square inch, and \( w_c \) ranges between
145 pcf for normal-density concrete and 100 pcf for structural lightweight concrete; \( f'_c \) in Equation 36.1b
and Equation 36.2b is in units of megapascals and \( w_c \) ranges between 2400 kg/m\(^3\) for normal-density
concrete and 1765 kg/m\(^3\) for lightweight concrete. The modulus of rupture of concrete can be taken as:

\[
f_r (\text{psi}) = 7.5\lambda \sqrt{f'_c} \\
f_r (\text{MPa}) = 0.632\lambda \sqrt{f'_c}
\]

where:

\( \lambda = 1.0 \) for normal-density stone aggregate concrete.
\( \lambda = 0.85 \) for sand lightweight concrete.
\( \lambda = 0.75 \) for all lightweight concrete.

#### 36.1.2 Creep of Concrete

Concrete creeps under sustained loading due to transverse flow of the material. The creep coefficient as
a function of time can be calculated from the following expression (ACI Committee 435, 1995; Nawy,
2002, 2008):

\[
C_t = \left(\frac{t^{0.6}}{10 + t^{0.6}}\right)C_u
\]

where time \( t \) is in days and \( C_u \), the ultimate creep factor, is 2.35. The short-term deflection is multiplied
by \( C_t \) to get the long-term deflection, which is added to the short-term (instantaneous) deflection value
to obtain the total deflection.
36.1.3 Shrinkage of Concrete

Concrete shrinks as the absorbed water evaporates and the chemical reaction of cement gel proceeds. For moist-cured concrete, the shrinkage strain that occurs at any time \( t \) in days 7 days after placing the concrete can be evaluated from (ACI Committee 435, 1995):

\[
\varepsilon_{\text{st}}(t) = \left(\frac{t}{35+t}\right)\varepsilon_{\text{st}}\text{a}
\]  

(36.5a)

For steam-cured concrete, the shrinkage strain at any time \( t \) (in days) 1 to 3 days after placement of the concrete is:

\[
\varepsilon_{\text{st}}(t) = \left(\frac{t}{55+t}\right)\varepsilon_{\text{st}}\text{a}
\]

(36.5b)

where maximum \( \varepsilon_{\text{st}}\text{a} \) can be taken as \( 780 \times 10^{-6} \) in./in. (mm/mm).

Shrinkage and creep due to sustained load can also be evaluated from the ACI expression (ACI Committee 318, 2008):

\[
\lambda = \left(\frac{\xi}{1+50\rho'}\right)
\]

(36.5c)

and Figure 36.1 for the factor \( \xi \) that ranges from a value of 2.0 for 5 years or more to 1.0 for 3 months of sustained loading; \( \rho' = \) compression steel percentage = \( A_s/\ell b d \).

36.1.4 Control of Deflection

Serviceability is a major factor in designing structures to sustain acceptable long-term behavior. Serviceability is controlled by limiting deflection and cracking in the members (ACI Committee 435, 1995). For deflection computation and control, the effective moment of inertia of a cracked section can be evaluated from the Branson equation:

\[
I_e = \left(\frac{M_o}{M_o}\right)I_g + \left[1 - \left(\frac{M_o}{M_o}\right)^3\right]I_o \leq I_g
\]

(36.6)
where, for reinforced-concrete beams:

\[ M_c = \text{cracking moment due to total load} = \frac{(f_L)l}{y_t} \]

\[ y_t = \text{distance from the neutral axis to the extreme tension fibers.} \]

\[ M_a = \text{maximum service load moment at the section under consideration.} \]

\[ I_g = \text{gross moment of inertia}. \]

In the case of prestressed concrete,

\[ \left( \frac{M_{cr}}{M_a} \right) = f_{st} - f_L \]

where:

\[ M_{cr} = \text{moment due to that portion of live load moment} \]

\[ M_a = \text{maximum service load (unfactored) live load moment.} \]

\[ f_{st} = \text{total calculated stress in the member}. \]

\[ f_L = \text{calculated stress due to live load}. \]

For long-term deflection, Figure 36.1 gives the required multipliers as a function of time.

### 36.1.5 Control of Cracking in Beams

Control of cracking in beams and one-way slabs can be made using the expression (ACI Committee 224, 2001):

\[ w_{max} (\text{in.}) = 0.076 \beta f_s \sqrt{d/A} \times 10^{-3} \]

where:

\[ w_{max} = \text{crack width (in.) (25.4 mm)}. \]

\[ \beta = \frac{(h - c)}{(d - c)}. \]

\[ d_t = \text{thickness of cover to the first layer of bars (in.).} \]

\[ f_s = \text{maximum stress in reinforcement at service load} = 0.60 f_y (\text{kips/in.}^2). \]

\[ A = \text{area of concrete in tension divided by number of bars (in.}^2) = bh/\gamma, \text{where } \gamma \text{ is the number of bars at the tension side}. \]

### 36.1.6 ACI 318 Code Provisions for Control of Flexural Cracking

From the author’s work and briefly reported in Nawy (2002, 2005), the spacing of the reinforcement is a major parameter in limiting the crack width. As the spacing is decreased through the use of larger numbers of bars, the area of the concrete envelopes surrounding the reinforcement increases. This leads to a larger number of narrower cracks. As the crack width becomes narrow enough within the values given in Table 36.1, corrosion effects on the reinforcement are considerably reduced. The current ACI provisions on crack control deal with this problem by limiting reinforcement spacing in reinforced-concrete beams and one-way slabs to the values obtained from the following expression for maximum allowable bar spacing:

\[ s = 15 \left( \frac{40,000}{f_y} \right) - 2.5c_t \]

but not greater than 12(40,000/$f_y$), where:

\[ f_y = \text{calculated stress in reinforcement at service load} = \text{unfactored moment divided by the steel area and the internal arm moment}; f_y \text{ is taken as } 2/3f_y (\text{psi}). \]

\[ c_t = \text{clear cover from the nearest surface in tension to the flexural tension reinforcement (in.).} \]

\[ s = \text{center-to-center spacing of flexural tension reinforcement (in.) closest to the tension face of the section}. \]
From these provisions, the maximum spacing for 60,000-psi (414-MPa) reinforcement = 12 \left\{\frac{36}{(0.6 \times 60)}\right\} = 12 \text{ in. (305 mm)}. The maximum spacing of 12 in. conforms with tests conducted by the author on more than 100 two-way action slabs; hence, this limitation on the distribution of flexural reinforcement in one-way slabs and wide-web reinforced-concrete beams is appropriate. In beams of normal web width in normal buildings, however, these provisions might not be as workable as controlling the crack width through the use of crack-width expressions to control the crack width within tolerable limits.

The SI expression for the value of reinforcement spacing in Equation 36.8b and \( f_s \) (MPa) is:

\[
\left(36.8c\right)
\]

\[
s (\text{mm}) = 380 \left(\frac{280}{f_s}\right) - 2.5 c_e
\]

but not to exceed \(300(252/f_s)\). For the usual case of beams with grade-420 reinforcement and 50-mm clear cover to the main reinforcement and with \( f_s = 252 \text{ MPa} \), the maximum bar spacing is 300 mm.

It should be stressed that these provisions are applicable to reinforced-concrete beams and one-way slabs in structures subject to normal environmental conditions. For other types of structures subject to aggressive environment such as sanitary structures, refer to ACI Committee 350 (2006), Nawy (2002, 2008), and Table 36.1 for values of tolerable crack widths in reinforced-concrete structures.

### Table 36.1 Tolerable Crack Widths

<table>
<thead>
<tr>
<th>Exposure Condition</th>
<th>Tolerable Crack Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry air or protective membrane</td>
<td>0.016 0.41</td>
</tr>
<tr>
<td>Humidity, moist air or soil</td>
<td>0.012 0.30</td>
</tr>
<tr>
<td>Deicing chemicals</td>
<td>0.007 0.18</td>
</tr>
<tr>
<td>Seawater and seawater spray; wetting and drying</td>
<td>0.006 0.15</td>
</tr>
<tr>
<td>Water-retaining structures (excluding non-pressure pipes)</td>
<td>0.004 0.10</td>
</tr>
</tbody>
</table>

© 2008 by Taylor & Francis Group, LLC
Concrete cylinder compressive strength.
\( f_y \) = yield strength of the reinforcement.
\( A_c \) = gross area of the concrete section.
\( A_s \) = area of the reinforcement.

The factor 0.85 representing the adjustment in concrete strength between the cylinder test result and the actual concrete strength in the structural element has been shown by extensive testing to be sufficiently accurate for higher strength concretes (ACI Committee 363, 1992; Nawy, 2008).

Confining the concrete in compression members through the use of spirals or closely spaced ties increases its compressive capacity. The increase in concrete strength due to the confining effect of the spirals can be represented by the following expression:

\[
\begin{align*}
\frac{f_c'}{f_c''} = & \frac{1}{4} \left[ \frac{f_c}{f_c''} - f_c'' \right] \\
& (36.10a)
\end{align*}
\]

where:
\( f_c' \) = concrete confining stress due to the spiral.
\( f_c'' \) = compressive strength of the confined concrete.
\( f_c''' \) = compressive strength of the unconfined column concrete.

The hoop tension force in the circular spiral is:

\[ 2A_s f_s = f_c' D_t' \]
or

\[ f'_c = \frac{2A_wf_y}{D'_s} \quad (36.10b) \]

where:

- \( A_w \) = cross-sectional area of the spiral.
- \( D'_s \) = diameter of concrete core.
- \( s \) = spiral pitch.

Equation 36.10 can be improved (ACI Committee 363, 1992; Nawy, 2002), leading to the following form for normal weight concrete:

\[ (f'_c - f'_c) = 4.0f'_c(1 - s/D'_s) \quad (36.11a) \]

and for lightweight concrete:

\[ (f'_c - f'_c) = 1.8f'_c(1 - s/D'_s) \quad (36.11b) \]

Figure 36.3 gives the results of peak stress comparisons vs. axial strain for spirally reinforced members for low-, medium-, and high-strength concretes. For higher strength, it shows a lower strain at peak load and a steeper decline past the peak value; however, the strength gain in concrete due to confinement seems to be well predicted for high-strength concretes in Equation 36.11.

### 36.2.2 Beams and Slabs

#### 36.2.2.1 The Compressive Block

The design of concrete structural elements is based on the compressive stress distribution across the depth of the member as determined by the stress–strain diagram of the material. For high-strength concretes, the difference in the shape of the stress–strain relationship discussed in connection with Figure 36.2 results in differences in the shape of the compressive stress block. Figure 36.4 shows possible compressive blocks for use in design. Figure 36.4c could more accurately represent the stress distribution.
Concrete Construction Engineering Handbook

for higher strength concrete; however, the computed strength of beams and eccentrically loaded columns depends on the reinforcement ratio. In the ACI 318 Code provisions, which use the equivalent rectangular block, the nominal moment strength of a singly reinforced beam is calculated using the following expression:

\[ M_n = A_f d \left[ 1 - 0.59 \frac{f_{cr}}{f_{cf}} \right] \]  

(36.12)

where the coefficient \( 0.59 = \frac{\beta_2}{\beta_1 \beta_3} \). Although a detailed evaluation of the factors \( \beta_1, \beta_2, \) and \( \beta_3 \) indicates a significant difference in their separate values, depending on the concrete strength (ACI Committee 363, 1992), Figure 36.5 shows that these differences collectively balance each other and that the combined coefficient \( \frac{\beta_2}{\beta_1 \beta_3} \) is well represented by the 0.59 value. Consequently, for strengths up to 12,000 psi (42

**FIGURE 36.4** Concrete compressive stress block: (a) standard stress block; (b) equivalent rectangular block; and (c) modified trapezoidal block.

**FIGURE 36.5** Stress block parameter \( \frac{\beta_2}{\beta_1 \beta_3} \) vs. concrete compressive strength. (From ACI Committee 363, State-of-the-Art on High-Strength Concrete, ACI 363R, American Concrete Institute, Farmington Hills, MI, 1992, pp. 1–55.)
MPa), the current ACI 318 Code expressions requiring that beams be under-reinforced are equally applicable. For considerably higher strengths or for members combining compression and bending or for the over-reinforced members allowed in the codes, some differences in the value of $\beta_2/\beta_1 \beta_3$ can be expected.

### 36.2.2.2 Compressive Limiting Strain

Although high-strength concrete achieves its peak value at a unit strain slightly higher than that of normal-strength concrete (Figure 36.2), the ultimate strain is lower for high-strength concrete unless confinement is provided. A limiting strain value allowed by the ACI 318 Code is 0.003 in./in. (mm/mm). Other codes allow a limiting strain for unconfined concrete of 0.0035 or 0.0038. The conservative ACI value of 0.003 seems to be adequate for high-strength concretes as well, although it is somewhat less conservative than that for lower strength concretes.

### 36.2.2.3 Confinement and Ductility

As higher strength concrete is more brittle, confinement becomes more important in order to increase its ductility. If $\mu$ is the deflection ductility index,

$$\mu = \frac{\Delta_u}{\Delta_y}$$  \hspace{1cm} (36.13)

where:

- $\Delta_u = \text{beam deflection at failure load.}$
- $\Delta_y = \text{beam deflection at the load producing yield of the tensile reinforcement.}$

Table 36.2 shows the ductility index values of concretes in singly reinforced beams ranging in strength from 3700 to 9265 psi (25 to 64 MPa). The corresponding reduction in the ductility index ranges from 3.54 to 1.07. The addition of compressive reinforcement and confinement to geometrically similar beams seems to increase the ductility index for $f'_c = 8500$ up to a value of 5.61. Hence, the higher the concrete compressive strength, the more it becomes necessary to provide for confinement or the addition of compression steel ($A'_s$) while using the same expression for nominal moment strength that is applicable to normal-strength concretes. It should be stated that cost would not be affected to any meaningful extent, as diagonal tension and torsion stirrups have to be used anyway, and in seismic regions closely spaced confining ties are a requirement. The maximum strain of confined concrete that can be utilized should not exceed 0.01 in./in. (mm/mm) in limit design.

### TABLE 36.2 Deflection Ductility Index for Singly Reinforced Beams

<table>
<thead>
<tr>
<th>Beam</th>
<th>$f'_c$</th>
<th>$\rho/\rho_b$</th>
<th>Ductility Index ($\mu = \Delta_u/\Delta_y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>3700</td>
<td>26</td>
<td>0.51</td>
</tr>
<tr>
<td>A2</td>
<td>6500</td>
<td>45</td>
<td>0.52</td>
</tr>
<tr>
<td>A3</td>
<td>8535</td>
<td>59</td>
<td>0.29</td>
</tr>
<tr>
<td>A4</td>
<td>8535</td>
<td>59</td>
<td>0.64</td>
</tr>
<tr>
<td>A5</td>
<td>9264</td>
<td>64</td>
<td>0.87</td>
</tr>
<tr>
<td>A6(a)</td>
<td>8755</td>
<td>60</td>
<td>1.11</td>
</tr>
</tbody>
</table>

* Ratio of tension reinforcement divided by reinforcement ratio producing balanced strain conditions.

36.2.2.4 Shear and Diagonal Tension

Design for shear in accordance with the ACI 318 Code is based on permitting the plain concrete in the web to assume part of the nominal shear $V_n$. If $V_c$ is the shear strength resistance of the concrete, the web stirrups resist a shear force $V_s = V_n - V_c$. High-strength concrete develops a relatively brittle failure, as previously discussed, with the aggregate interlock decreasing with the increase in the compressive strength. Hence, the shear friction and diagonal tension failure capacity in beams might be unconservatively represented by the ACI 318 equations (Ahmed and Lau, 1987); however, the strength of the diagonal struts in the beam shear truss model is increased through the mobilization of more stirrups and the increased load capacity of the struts themselves. No research data are currently available to provide definitive guidelines on the minimum web steel that can prevent brittle failure. All work to date indicates no unsafe use of the current ACI 318 Code provisions for shear in the design of high-strength concrete members.

36.3 Strength Design of Reinforced-Concrete Members

36.3.1 Strain Limits Method for Analysis and Design

This approach is sometimes referred to as the unified method, as it is equally applicable to flexural analysis of prestressed concrete elements. The nominal flexural strength of a concrete member is reached when the net compressive strain in the extreme compression fibers reaches the ACI 318 Code strain limit of 0.003 in./in. It also stipulates that when the net tensile strain in the extreme tension steel ($\varepsilon_t$) is sufficiently large at a value equal or greater than 0.005 in./in., the behavior is fully ductile. The concrete beam section under this condition is characterized as being tension controlled, with ample warning of failure as denoted by excessive cracking and deflection.

If the net tensile strain at the extreme tension steel ($\varepsilon_t$) is small, such as in compression members, being equal or less than a compression-controlled strain limit, a brittle mode of failure is expected, with little warning of such an impending failure. Flexural members are usually tension controlled, and compression members are usually compression controlled; however, in some sections, such as those subjected to small axial loads but large bending moments, the net tensile strain ($\varepsilon_t$) in the extreme tensile reinforcement will have an intermediate or transitional value between the two strain limit states—namely, between the compression-controlled strain limit of $\varepsilon_t = f_y/E_s = 60,000/29 \times 10^6 = 0.002$ in./in., and the tension-controlled strain limit $\varepsilon_t = 0.005$ in./in. Figure 36.6 illustrates these three zones as well as the variation in the strength reduction factors applicable to the total range of behavior.

For the tension-controlled state, the strain limit $\varepsilon_t = 0.005$ corresponds to the reinforcement ratio $\rho/\rho_b = 0.63$, where $\rho_b$ is the reinforcement ratio for the balanced strain $\varepsilon_t = 0.002$ in the extreme tensile reinforcement for 60-ksi steel. The net tensile strain $\varepsilon_t = 0.005$ for a tension-controlled state is a single value that applies to all types of reinforcement, whether mild steel or prestressing steel. High reinforcement ratios that produce a net tensile strain less than 0.005 result in a $\phi$-factor value lower than 0.90, resulting in less economical sections. Therefore, it is more efficient to add compression reinforcement if necessary or to deepen the section to make the strain in the extreme tension reinforcement ($\varepsilon_t$) $\geq 0.005$. Variation of the strain reduction factor $\phi$ as a function of strain for the range values of $\varepsilon_t = 0.002$ and $\varepsilon_t = 0.005$ can be linearly interpolated from the following expressions in terms of the limit strain $\varepsilon_t$:

**Tied sections:**

$$0.65 \leq \left[ \phi = 0.65 + (\varepsilon_t - 0.002) \left( \frac{250}{3} \right) \right] \leq 0.90$$

(36.14a)

**Spirally reinforced sections:**

$$0.75 \leq \left[ \phi = 0.75 + (\varepsilon_t - 0.002) \left( \frac{150}{3} \right) \right] \leq 0.90$$

(36.14b)
Variation of $\phi$ as a function of the neutral axis depth ratio $c/d_t$ can be evaluated from the following two expressions for the limit ratios of $c/d_t$ of 0.60 for the compression-controlled state and 0.375 for the tension-controlled state:

**Tied sections:**

$$0.65 \leq \phi = 0.65 + 0.25 \left( \frac{1}{c/d_t} - \frac{5}{3} \right) \leq 0.90$$  
(36.15a)

**Spirally reinforced sections:**

$$0.75 \leq \phi = 0.75 + 0.15 \left( \frac{1}{c/d_t} - \frac{5}{3} \right) \leq 0.90$$  
(36.15b)

### 36.3.2 Flexural Strength

#### 36.3.2.1 Singly Reinforced Beams

Flexural strength is determined from the strain and stress distribution across the depth of the concrete section. Figure 36.7 shows the stress and strain distribution and forces. Taking moments of all the forces about tensile steel ($A_t$) gives, for singly reinforced beams ($A'_t = 0$), a nominal moment strength:

$$M_n = A_t f_y \left( d - \frac{a}{2} \right)$$  
(36.16)

$$M_n = bd^2 \omega (1 - 0.59\omega)$$  
(36.17)

where $\omega =$ reinforcement index $= A_t/bd \times f_y/f'_c$. 

© 2008 by Taylor & Francis Group, LLC
FIGURE 36.7 Stress and strain distribution across beam depth: (a) beam cross-section; (b) strain across depth; (c) actual stress block; and (d) assumed equivalent stress block.
The balanced strain-state reinforcement ratio \( \rho_b \) for simultaneous yielding of the reinforcement at the tension side and crushing of the concrete at the compression side can be obtained from the following expression:

\[
\rho_b = 0.85 \beta_1 \left( \frac{f_s}{E_s} \right) \left( \frac{\varepsilon_c}{0.004} \right)
\]

where \( \beta_1 = 0.85 \) and reduces at the rate of 0.05 per 1000 psi in excess of 4000 psi, namely:

\[
\beta_1 = 0.85 - 0.05 \left( \frac{f_s' - 1000}{1000} \right)
\]

with a minimum \( \beta_1 \) value of 0.65.

### 36.3.2.2 Doubly Reinforced Beams

For doubly reinforced sections that have compression steel that yielded:

\[
M_n = (A_s - A_s') f_s \left( y - \frac{a}{2} \right) + A_s' f_s' (d - d')
\]

If compressive reinforcement is used in a doubly reinforced section as in Figure 36.8 for compression members, the depth of the compressive block is:

\[
a = \frac{A_s f_s - A_s' f_s'}{0.85 f_b}
\]

where \( b \) is the width of the section at the compression side, and \( f_s' \) is the stress in the compression.
36.3.2.3 Flanged Sections

For flanged sections where the neutral axis falls outside the flange:

\[ M_u = (A_s - A_d) f_y \left( d - \frac{a}{2} \right) + A_d f_y \left( d - \frac{h_f}{2} \right) \]  \hspace{1cm} (36.21)

where:

\[ A_d = \left[ 0.85 f'c (b - b_w) h_f \right] / f_y \]

\[ b_w = \text{web width}. \]

\[ h_f = \text{flange thickness}. \]

The depth is:

\[ a = \frac{A_s f_y}{0.85 f'y} > h_f \]  \hspace{1cm} (36.22)

\[ \rho_f = 0.85 f'(b - b_w) \frac{h_f}{f_y b_w d} \]

36.3.2.4 Minimum Reinforcement

The flexural reinforcement percentage \( \rho \) has to have a minimum value of \( \rho_{min} = 3 \sqrt{f_c' / f_y} \) for positive moment reinforcement, and \( \rho_{min} = 6 \sqrt{f_c' / f_y} \) for negative moment reinforcement but always not less than \( 200 / f_y \), where \( f_y \) is in units of pounds per square inch. The factored moment is:

\[ M_u = \phi M_u \]  \hspace{1cm} (36.23)

where \( \phi = 0.90 \) for flexure.

36.3.3 Shear Strength

External transverse load is resisted by internal shear to maintain section equilibrium. As concrete is weak in tension, the principal tensile stress in a beam cannot exceed the tensile strength of the concrete. The principal stress is composed of two components: shear stress and flexural stress. It is important that the beam web be reinforced to prevent diagonal shear cracks from opening. The resistance of the plain concrete in the web sustains part of the shear stress, and the balance has to be borne by the diagonal tension reinforcement. The shear resistance of the plain concrete is known as the nominal shear strength, or \( V_c \):

\[ V_c = 2.0 \lambda \sqrt{f_c' b_w d} \text{ (lb)} \leq 3.5 \lambda \sqrt{f_c' } \text{ (lb)} \]  \hspace{1cm} (36.24a)

\[ V_c = \left( \frac{\lambda \sqrt{f_c'}}{6} \right) b_w d \text{ (N)} \]  \hspace{1cm} (36.24b)

or

\[ V_c = \left[ 1.9 \lambda \sqrt{f_c'} + 2500 \rho_{w} \frac{V_c d}{M_u} \right] b_w d \text{ (lb)} \leq 3.5 \lambda \sqrt{f_c' } \text{ (lb)} \]  \hspace{1cm} (36.25a)

\[ V_c = \left[ \left( \frac{\lambda \sqrt{f_c'}}{7} + 120 \rho_{w} \frac{V_c d}{M_u} \right) / 7 \right] b_w d \text{ (N)} \]  \hspace{1cm} (36.25b)
Values for $\lambda$ are given in Section 36.1.

$$\rho_w = \frac{A_v}{b_w d} \quad \text{and} \quad \frac{V_u d}{M_u} \leq 1.0$$

No web steel is needed if $V_u < 1/2 V_c$. For calculating $V_u$, the critical section is at a distance $d$ from the support face. Spacing of the web stirrups is as follows:

$$s = \frac{A_v f_y d}{V_u / \phi - V_i}$$

(36.26)

where $A_v$ is the cross-sectional area of web steel, and $\phi$ is 0.85 for shear and torsion. The transverse web steel is designed to carry the shear load $V_s = V_u - V_i$. The spacing of the stirrups is governed by the following:

$$V_i \geq 4\lambda \sqrt{f_y} : s = \frac{d}{4}$$

$$V_i \leq 2\lambda \sqrt{f_y} : s = \frac{d}{2}$$

$$V_i \geq 8\lambda \sqrt{f_y} : \text{enlarge section}$$

The minimum sectional area of the stirrups is $A_{v,\min} = 50b_w s f_y / s_{max} = d/2$ where shear is to be considered; $d$ is the effective depth to the center of the tensile reinforcement; and $f_y$ is the yield strength of the steel (lb/in.$^2$).

### 36.3.4 Strut-and-Tie Theory and Design of Corbels and Deep Beams

#### 36.3.4.1 Strut-and-Tie Mechanism

As an alternative to the usual approach where plane sections before bending are considered to remain plane after bending, the strut-and-tie model is applied effectively in regions of discontinuity. These regions could be the support sections in a beam, the zones of load application, or the discontinuity caused by abrupt changes in section, such as brackets, beam daps, pile caps cast with column sections, portal frames, and others. Consequently, structural elements can be divided into segments called $B$-regions, where the standard beam theory applies with the assumption of linear strains, and $D$-regions, where the plane sections hypothesis is no longer applicable.

The analysis essentially follows the truss analogy approach, where parallel inclined cracks are assumed and expected to form in the regions of high shear. The concrete between the inclined cracks carries inclined compressive forces acting as diagonal compressive struts. The provision of transverse stirrups along the beam span results in a truss-like action where the longitudinal steel provides the tension chord of the truss as a tie, hence the “strut-and-tie” expression. Depending on the interpretation of the designer, simplifications of the paths of forces that are chosen to represent the real structure can considerably differ; consequently, this approach is more an art than an engineering science in the selection of the models, and significant over-design and serviceability checks become necessary. For equilibrium, at least three forces have to act at a joint, termed the *node*. Figure 36.9 demonstrates the simplified truss model for simply supported deep beams loaded on the top fibers, and Figure 36.10 shows a continuous beam model, as presented in the ACI 318 Code, both outlining the compression struts, the tension ties, and the nodes at the D regions which are the points of load application, discontinuity, and the support regions. Naturally, other possible alternative models can also be used, provided that they satisfy equilibrium and compatibility.
36.3.4.2 ACI Design Requirements by the Strut-and-Tie Method

Nodal Forces:

\[ \phi F_n \geq F_{nu} \]  
\[ (36.27) \]

where:

- \( F_n \) = nominal strength of a strut, tie, or nodal zone (lb).
- \( F_{nu} \) = factored force acting on a strut, tie, bearing area, or nodal zone (lb).
- \( \phi \) = for both struts and ties (similar to the strength reduction for shear).

Strength of Struts

\[ F_{ns} = f_{ce} A_{cs} \]  
\[ (36.28) \]

where:

- \( F_{ns} \) = nominal strength of strut (lb).
- \( A_{cs} \) = effective cross-sectional area at one end of a strut, taken perpendicular to the axis of the strut (in\(^2\)).
- \( f_{ce} \) = effective compressive strength of the concrete in a strut or nodal zone (psi).

\[ f_{ce} = 0.85\beta f'_c \]  
\[ (36.29) \]

where:

- \( \beta = 1.0 \) for struts that have the same cross-sectional area of the mid strut cross-section in the case of bubble struts.
- \( \beta = 0.75 \) for struts with reinforcement resisting transverse tensile forces.
- \( \beta = 0.40 \) for struts in tension members or tension flanges.
- \( \beta = 0.60 \) all other cases.
Longitudinal Reinforcement

\[ F_{wa} = f'_{c} A_{c} + A'_{s} f'_{s} \]  

(36.30)

where:

- \( A_{c}' \) = area of compression reinforcement in a strut (in.\(^2\)).
- \( f'_{c} \) = stress in compression reinforcement (in.\(^2\)).

Strength of Ties

\[ F_{nt} A_{nt} f_{y} + A_{ps} \left( f_{ps} + \Delta f_{ps} \right) \]  

(36.31)

where:

- \( F_{nt} \) = nominal strength of tie (lb).
- \( A_{nt} \) = area of non-prestressed reinforcement in a tie (in.\(^2\)).
- \( A_{ps} \) = area of prestressing reinforcement (in.\(^2\)).
- \( f_{ps} \) = effective stress after losses in prestressing reinforcement.
- \( \Delta f_{ps} \) = increase in prestressing stress beyond the service load level.
- \( f_{ps} + \Delta f_{ps} \) should not exceed \( f_{ps} \).

When no prestressing reinforcement is used, \( A_{ps} = 0 \) in Equation 36.31.
where \( h_{\text{e,max}} \) is the maximum effective height of concrete concentric with the tie, used to dimension the nodal zone (in.). If the bars in the tie are in one layer, the effective height of the tie can be taken as the diameter of the bars in the tie plus twice the cover to the surface of the bars. The reinforcement in the ties has to be anchored by hooks, mechanical anchorages, post-tensioning anchors, or straight bars, all with full development length.

**Strength of Nodal Zones**

\[
F_{nn} = f_{ce} A_n
\]  
(36.33)

where:

\( F_{nn} = \) nominal strength of a face of a nodal zone (lb).

\( A_n = \) area of the face of a nodal zone or a section through a nodal zone (in.\(^2\)).

It can be assumed that the principal stresses in the struts and ties act parallel to the axes of the struts and ties. Under such a condition, the stresses on faces perpendicular to these axes are principal stresses.

**Confinement in the Nodal Zone**

The ACI 318 Code stipulates that, unless confining reinforcement is provided within the nodal zone and its effect is supported by analysis and experimentation, the computed compressive stress on a face of a nodal zone due to the strut and tie forces should not exceed the values given by Equation 36.34:

\[
f_{ce} = 0.85 \beta_n f' c
\]  
(36.34)

where:

\( \beta_n = 1.0 \) in nodal zones bounded by struts or bearing stresses.

\( \beta_n = 0.8 \) in nodal zones anchoring one tie.

\( \beta_n = 0.6 \) in nodal zones anchoring two or more ties.

For detailed design examples, refer to Nawy (2008).

### 36.3.4.3 Design Example of a Corbel by the Strut-and-Tie Method

Design a corbel to support a factored vertical load \( V_u = 80,000 \) lb (160 kN) acting at a distance \( a_v = 5 \) in. (127 mm) from the face of the column. It has width \( b = 10 \) in. (254 mm), total depth \( h = 18 \) in. (457 mm), and an effective depth \( d = 14 \) in. (356 mm) (Nawy, 2008). Given:

\( f' c = 5000 \) psi (34.5 MPa) for normal weight concrete.

\( f_y = f_{yt} = 60,000 \) psi (414 MPa).

The supporting column size is 12 × 14 in. Assume the corbel is to be monolithically cast with the column, and neglect the weight of the corbel in the computations.

**Solution**

1. Assume that the corbel is monolithically cast with the column. Total depth \( h = 18 \) in. and effective depth \( d = 14 \) in. are based on the requirement that the vertical dimension of the corbel outside the bearing area must be at least one half the column face width of 14 in. (column size, 12 × 14 in.). Select a simple strut-and-tie model as shown in Figure 36.11, assuming that the center of tie AB is located at a distance of 4 in. below the top extreme corbel fibers, using one layer of reinforcing bars. Also assume that horizontal tie DG lies on a horizontal line passing at the re-entrant corner C of the corbel. The solid lines in Figure 36.11 denote tension tie action (T), and the dashed lines denote compression strut action (C). The nodal points A, B, C, and D result from the selected strut-and-tie model. Note that the entire corbel is a D-region structure because of the existing statics discontinuities in the geometry of the corbel and the vertical and horizontal loads.
2. The strut and tie truss forces are $N_{cu} = 0.20$ and $V_u = 16,000$ lb. The following are the truss member forces calculated from statics in Figure 36.11.

**Compression strut BC:**

Length $BC = \sqrt{(7)^2 + (14)^2} = 15.652$ in.

$$F_{BC} = 80,000 \times \frac{15.652}{14} = 89,443 \text{ lb}$$

**Tension tie BA:**

$$F_{BA} = 80,000 \times \frac{7}{14} + 16,000 = 56,000 \text{ lb}$$
Concrete Construction Engineering Handbook

Compression strut AC:

\[ F_{AC} = \frac{56,000\sqrt{(8)^2 + (14)^2}}{8} = 112,872 \text{ lb} \]

Tension tie AD:

\[ F_{AD} = 112,872 \times \frac{14}{\sqrt{(8)^2 + (14)^2}} = 98,000 \text{ lb} \]

Compression strut CC:

\[ F_{CC} = 80,000 + 98,000 = 178,000 \text{ lb} \]

Tension tie CD:

\[ F_{CD} = 56,000 - 40,000 = 16,000 \text{ lb} \]

Steel bearing plate design:

\[ f_u = \phi (0.85 f'c), \text{ where } \phi = 0.75 \text{ for bearing in strut-and-tie models.} \]

Area of plate is

\[ A_{f,ts} = \frac{80,000}{0.75(0.85 f'c)} = \frac{80,000}{0.75 \times 0.85 \times 5000} = 25.10 \text{ in.}^2 \]

Use a 5-1/2 × 5-1/2-in. plate and select a thickness to produce a rigid plate.

Tie reinforcement design:

\[ A_{u,AB} = \frac{56,000}{0.75 \times 60,000} = 1.25 \text{ in.}^2 \]

Use three #6 bars = 1.32 in.\(^2\) or, conservatively, three #7 bars = 1.80 in.\(^2\). These top bars in one layer have to be fully developed along the longitudinal column reinforcement:

\[ A_{u,CD} = \frac{16,000}{0.75 \times 60,000} = 0.36 \text{ in.}^2 \]

Use two #6 tie bars = 0.88 in.\(^2\) to form part of the cage shown in Figure 36.12.

Horizontal reinforcement \( A_h \) for crack control of shear cracks:

\[ A_h = 0.50(A_u - A_n) \]

where \( A_n \) is the reinforcement resisting the frictional force \( N_{ser} \)

\[ A_n = \frac{N_{ser}}{0.75 \times 60,000} = 0.36 \text{ in.}^2 \]

Hence, \( A_h = 0.50(1.25 - 0.36) = 0.45 \text{ in.}^2 \). Three #3 closed ties, evenly spaced vertically as shown in Figure 36.12, give \( A_h = 3(2 \times 0.11) = 0.66 \text{ in.}^2 > 0.45 \text{ in.}^2 \). Because \( \beta = 0.75 \) is used for calculating the effective concrete compressive strength in the struts in the following section, where \( f_{cu} = 0.85 \beta f'c \), the minimum reinforcement provided also has to satisfy:

\[ \sum \frac{A_h / \text{tie}}{b_{ti}} \sin \gamma_i \geq 0.003 = \frac{2(0.11)}{14 \times 3.0} \sin 60^\circ 15' = 0.0045 > 0.003 \]

Hence, adopt three #3 closed ties at 3.0-in. center-to-center spacing.
Proportioning Concrete Structural Elements by the ACI 318-08 Code

Strut capacity evaluation:

Strut CC—The width (\(w_s\)) of nodal zone C has to satisfy the allowable stress limit on the nodal zones, namely node B below the bearing plate and node C in the re-entrant corner to the column. Both nodes are considered unconfined.

\[ F_{SCC} \text{ for } 10\left(\frac{w_s}{2}\right) = 80\times(5+10)+16\times18 \text{ to give } \frac{w_s}{2} = 1.64 \text{ in.} \]

hence, strut width \(w_s = 3.28\) in. fits within the available concrete dimension about the strut center line.

Strut BC—Nominal strength is limited to \(F_{nc} = f_{ce}A_{sc}\), where \(f_{ce} = 0.85\beta_s f_{sc}'\); thus, \(f_{ce} = 0.85 \times 0.75 \times 5000 = 3188\) psi = 3.188 ksi. \(A_{sc}\) is the smaller strut cross-sectional area at the two ends of the strut, namely at node C, while at node B, the node width can be assumed equal to the steel plate width of 5.50 in. \(A_s\) at node C = 14 \times 3.28 = 45.92 in.\(^2\). Available factored \(F_{ncC} = \Phi F_{ncC} = 0.75 \times 3.188 \times 45.92 = 109.8\) kip, which is greater than the required \(F_{BC} = 89.4\) kip.

Strut AC

Required width (\(w_s\)) of strut \(A_j = \frac{F_{SCC}}{\Phi f_{ce} b} = \frac{112.87 \text{ kip}}{0.75 \times 3.188 \times 14} = 3.37 \text{ in.} \)

Examination of the corbel and the column depth of 12 in. suggest a minimum clear cover of 2.0 in. from the outer concrete surface; hence, the widths (\(w_s\)) of all struts fit within the corbel geometry. Adopt the design as shown in Figure 36.12.
36.3.5 Torsional Strength

The space truss analogy theory is used for the analysis and design of concrete members subjected to torsion. It is based on the shear flow in a hollow tube concept and the summation of the forces in the space truss elements (ACI Committee 318, 2008; Hsu, 1993; Nawy, 2002, 2006, 2008). ACI Committee 318 stipulates disregarding the concrete nominal strength $T_c$ in torsion and assigning all the torque to the longitudinal reinforcement ($A_\ell$) and the transverse reinforcement ($A_t$), essentially assuming that the volume of the longitudinal bars is equivalent to the volume of the closed transverse hoops or stirrups. The critical section is taken at a distance $d$ from the face of the support for the purpose of calculating torque $T_u$.

Sections that are subjected to combined torsion and shear should be designed for torsion if the factored torsional moment ($T_u$) exceeds the following value for nonprestressed members:

$$T_u > \phi \lambda \sqrt{f'_c \left( \frac{A_{up}^2}{P_{op}} \right)}$$  \hspace{1cm} (36.35)

where:

- $A_{up} = \text{area enclosed by the outside perimeter of the concrete cross-section.}$
- $P_{op} = \text{outside perimeter of the cross-section (}A_{up}\text{)} \text{ (in.).}$

Two types of torsion are considered: (1) equilibrium torsion, where no redistribution of torsional moment is possible—in this case, all the factored torsional moment is designed for; and (2) compatibility torsion, where redistribution of the torsional moment occurs in a continuous floor system—in this case, the maximum torsional moment to be provided for is:

$$T_u = \phi 4\lambda \sqrt{f'_c \left( \frac{A_{up}^2}{P_{op}} \right)}$$  \hspace{1cm} (36.36)

The concrete section has to be enlarged if:

$$\sqrt{\left( \frac{V_u}{b_u d} \right)^2 + \left( \frac{T_u p_h}{1.7 A_{oh}} \right)^2} \geq \phi \left( \frac{V_u}{b_u d} \right) + 8\lambda \sqrt{f'_c}$$  \hspace{1cm} (36.37)

where:

- $p_h = \text{perimeter of centerline of outermost closed transverse torsional reinforcement (in.).}$
- $A_{oh} = \text{area enclosed by centerline of the outermost closed transverse torsional reinforcement (in.}^2\text{).}$

The transverse torsional reinforcement should be chosen with such size and spacing $s$ that:

$$A_t = \frac{T_u}{2A_{sp} f_y \cot \theta}$$  \hspace{1cm} (36.38)

where:

- $A_{sp} = \text{gross area enclosed by the shear path} = 0.85 A_{oh}$
- $\theta = \text{angle of compression diagonals (45° in reinforced concrete, 37.5° in prestressed concrete)}.$

The longitudinal torsional reinforcement ($A_\ell$) divided equally along the four faces of the beam is:

$$A_\ell = \frac{A_t}{s} p_h \left( \frac{f_{yr}}{f_{yt}} \right) \cot^2 \theta$$  \hspace{1cm} (36.39)

$$A_{\ell, \min} = \frac{5 \sqrt{f'_c A_{up}}}{f_{yt}} - \left( \frac{A_t}{s} \right) p_h \frac{f_{yr}}{f_{yt}}$$  \hspace{1cm} (36.40)
where:
\[ f_{yv} = \text{yield strength of the transverse reinforcement}. \]
\[ f_{yl} = \text{yield strength of the longitudinal reinforcement}. \]

The minimum area of transverse reinforcement is:

\[ A_v + 2A_t \geq \frac{50b_v s}{f_y} \quad (36.41) \]

Maximum \( s \) is 12 in.

In SI units, the following are equivalent expressions:

Equation 36.35:
\[ T_v = \frac{\phi \lambda \sqrt{f_y}}{12} \left( \frac{A_p^2}{P_p} \right) \]

Equation 36.36:
\[ T_v = \frac{\phi \lambda \sqrt{f_y}}{3} \left( \frac{A_p^2}{P_p} \right) \]

For Equation 36.37, the right-hand expression is:

\[ \phi \left( \frac{V_v}{b_v d} \right) + \left( \frac{8 \lambda \sqrt{f_y}}{12} \right) \text{MPa} \]

For Equation 36.40:
\[ A_{t,\text{min}} = \frac{5\sqrt{f_y A_p}}{12 f_{yl}} \left( \frac{A_v}{s} p_h \left( \frac{f_{yw}}{f_{yl}} \right) \right) \]

For Equation 36.41:
\[ A_v + 2A_t \geq \frac{0.35b_v s}{f_y} \]

where \( f_y \) is in megapascals. Maximum \( s \) is 300 mm.

### 36.3.6 Compression Members: Columns

#### 36.3.6.1 Nonslender Columns

Columns normally fall within the compression-controlled zone of Figure 36.6—namely, in the strain limit condition of \( \varepsilon_t = 0.002 \) or less at the extreme tension steel reinforcement level. The three modes of failure at the ultimate load state in columns can be summarized as follows:

- Tension-controlled state, by the initial yielding of the reinforcement at the tension side at \( c/d_i = 0.375 \)
- Transition state denoted by the initial yielding of the reinforcement at the tension side but with a strain \( \varepsilon_t \), less than 0.005 but greater than the balancing strain \( \varepsilon_t = 0.002 \) for Grade 60 steel, or \( \varepsilon_t = f_y/E \) for other reinforcement grades
- Compression-controlled state by initial crushing of the concrete at the compression face, where the balanced strain state occurs when failure develops simultaneously in tension and compression, a condition defined by the strain state \( \varepsilon_t = \varepsilon_y \) at the tension reinforcement at a strain level \( \varepsilon_t = 0.002 \) or less for Grade 60 steel
If \( P_{nb} \) is the axial load corresponding to the balanced limit strain condition—namely, when concrete at the compression face crushes simultaneously with the yielding of the extreme reinforcement at the tension face, then the modes of failure at ultimate load can also be defined as follows, where \( e_b \) is the eccentricity of the load at the balanced strain condition:

\[
\begin{align*}
P_n &< P_{nb}, \text{ tension failure (}e > e_b) \\
P_n &= P_{nb}, \text{ balanced failure (}e = e_b) \\
P_n &> P_{nb}, \text{ compression failure (}e < e_b)
\end{align*}
\]

In all of these cases, the strain-compatibility relationship must be maintained at all times through computation of the strain \( \varepsilon_s' \) in the compression side reinforcement on the basis of linearity of distribution of strain across the concrete section depth. It should be noted that, for each limit strain case, there are unique values of nominal thrust \( P_n \) and nominal moment \( M_n \). Consequently, a unique eccentricity \( e = M_n/P_n \) can be determined for each case. The expressions for load and moment for the balanced strain condition are:

\[
\begin{align*}
P_{nb} &= 0.85f_{bt}b_a + A_f' \varepsilon_s' - A_f f_y' \\
M_{nb} &= P_{nb}c_b = 0.85f_{bt}b_a \left( y - \frac{d_1}{2} \right) + A_f' \varepsilon_s' \left( y - d' \right) + A_f f_y \left( d - y \right)
\end{align*}
\]

where:

\[
f_y' = E_e \left[ \frac{0.003(c - d')}{c} \right] \leq f_y
\]

The force \( P_n \) and the moment \( M_n \) at the ultimate for any other eccentricity level are:

\[
\begin{align*}
P_n &= 0.85f_{bt}b_a + A_f' \varepsilon_s' - A_f f_y \\
M_n &= P_n e = 0.85f_{bt}b_a \left( \bar{y} - \frac{a}{2} \right) + A_f' \varepsilon_s' \left( \bar{y} - d' \right) + A_f f_y \left( d - \bar{y} \right)
\end{align*}
\]

where:

\[
f_y = E_e \left[ \frac{0.003(d - c)}{c} \right] \leq f_y
\]

and

\[
c = \text{ depth to the neutral axis.} \\
\bar{y} = \text{ distance from the compression extreme fibers to the center of gravity of the section.} \\
a = \text{ depth of the equivalent rectangular block = } \beta_1 c, \text{ where } \beta_1 \text{ is defined in Equation 36.18b.}
\]

The geometry of the compression member section and the forces acting on the section are shown in Figure 36.7. Equation 36.45 and Equation 36.46 are obtained from the equilibrium of forces and moments.

### 36.3.6.2 Slender Columns

If the compression member is slender—namely, the slenderness ratio \( kl/r \) exceeds 22 for unbraced members and \((34 - 12 \times M_1/M_2) \) for braced members—then failure will occur by buckling and not by material failure. In such a case, if \( kl/r \) is less than 100, then a first-order analysis such as the moment magnification method can be performed. If \( kl/r > 100 \), then the \( P - \Delta \) effects have to be considered and a second-order analysis has to be performed. The latter is a lengthy process and is more reasonably executed using readily available computer programs.
Moment Magnification Solution ($kl/r < 100$)

The larger moment $M_2$ is magnified such that:

$$M_c = \delta_m M_2$$  \hspace{1cm} (36.48)

where $\delta_m$ is the magnification factor. The column is then designed for a moment $M_c$ as a nonslender column. The subscript $ns$ is non-sidesway; $s$ is sidesway:

$$\delta_m = \frac{C_m}{1-(\frac{P_r}{0.75P_i})} \geq 1.0$$  \hspace{1cm} (36.49)

$$P_i = \frac{\pi^2 EI}{(k \ell_u)^2}$$  \hspace{1cm} (36.50)

$EI$ should be taken as:

$$EI = \frac{0.2E_c I_s + E_s I_u}{1+\beta_d}$$  \hspace{1cm} (36.51)

or

$$EI = \frac{0.4E_c I_s}{1+\beta_d}$$  \hspace{1cm} (36.52)

$$C_m = 0.6 + 0.4 \frac{M_i}{M_2} \geq 0.4$$  \hspace{1cm} (36.53)

If there is side-sway,

$$C_m = 0.6 + 0.4 \frac{M_i}{M_2} \geq 0.4$$  \hspace{1cm} (36.54a)

$$M_2 = M_{2n} + \delta_s M_{2n}$$  \hspace{1cm} (36.54b)

where:

$$\delta_s M_i = \frac{M_i}{\frac{\Sigma P}{0.75P_i}} \geq M_i$$  \hspace{1cm} (36.55a)

$$\delta_s M_i = \frac{M_i}{1-Q} \geq M_i$$  \hspace{1cm} (36.55b)

where:

$$\text{Stability index } Q = \frac{\Sigma P \Delta_s}{V_0 l_c} \leq 0.05$$  \hspace{1cm} (36.55c)

and

$\Delta_s$ = first-order relative lateral deflection between the top and bottom of that story due to factored horizontal total shear ($V_0$) of that story.

$l_c$ = length of compression member.

The non-sway moment $M_{2ns}$ is unmagnified, provided that the maximum moment is along the column height and not at its ends; otherwise, its value has to be multiplied by the non-sway magnifier $\delta_m$. If the stability index exceeds a value of 0.05, a second-order analysis becomes necessary. Effective length factor $k$ when there is single curvature can be obtained from Figure 36.13a. For double curvature, length factor $k$ can be obtained from Figure 36.13b. Discussion of the P-delta effect and the second-order analysis is given in Nawy, 2008).
36.3.7 Two-Way Slabs and Plates

Methods for designing two-way concrete slabs and plates include:

- ACI direct design method
- ACI equivalent frame method where effects of lateral loads can be considered
- Yield line theory
- Strip method
- Elastic solutions

FIGURE 36.13 Slender columns end effect factor $k$: (a) nonsway frames; (b) sway frames. (From ACI Committee 318, *Building Code Requirements for Structural Concrete*, ACI 318-08; Commentary, ACI 318R-08, American Concrete Institute, Farmington Hills, MI, 2008.)
The subject is too extensive to cover in this overview; however, the important concept of serviceability as controlled by deflection and cracking limitation is briefly presented.

### 36.3.7.1 Deflection Control

The thickness of two-way slabs for deflection control should be determined as follows:

**Flat Plate**

Use Table 36.3.

**Slab on Beams**

If $\alpha_m \leq 0.2$, use:

$$\alpha_m > 0.2 < 2.0, \quad h \geq \frac{t_n \left(0.8 + f_y / 200,000\right)}{36 + 5\beta(\alpha_m - 0.2)}$$  \hspace{1cm} (36.56)

but slab or plate thickness cannot be less than 5.0 in., so:

$$\alpha_m > 2.0, \quad \frac{t_n \left(0.8 + f_y / 200,000\right)}{36 + 9\beta}$$  \hspace{1cm} (36.57)

where:

- $\alpha_m =$ average value of $\alpha$ for all beams on edges of a panel.
- $\alpha = \frac{\text{flexural stiffness of beam section}}{\text{flexural thickness of slab width bounded laterally by the center line of the adjacent panels on each side of the beam}}$.
- $\beta =$ aspect ratio (long span/short span).

### 36.3.7.2 Crack Control

For crack control in two-way slabs and plates, the maximum computed weighted crack width due to flexural load (ACI Committee 224, 2001; Nawy, 2008) is as follows, where the parameter under the radical is the grid index ($G_i$):

$$w_{\text{max}} \text{ (in.)} = k\beta f_y \sqrt{\frac{\sqrt{S_d f_y}}{d_b} \frac{8}{\pi}}$$  \hspace{1cm} (36.58)

For $w_{\text{max}}$ (mm), multiply Equation 36.58 by 0.145 and use megapascals for $f_y$. Also,

- $k =$ fracture coefficient.
  - $2.8 \times 10^{-5}$ for a square uniformly loaded slab.
  - $2.1 \times 10^{-5}$ when the aspect ratio of short span/long span $< 0.75$ but $> 0.5$, or for a concentrated load.
  - $1.6 \times 10^{-5}$ for aspect ratio less than 0.5.
- $\beta = 1.25 = (h - c)/(d - c)$, where $c =$ depth to neutral axis.
- $f_y = 0.40 f_y$ (kip/in²).
36-28

Concrete Construction Engineering Handbook

\( h = \) total slab or plate thickness.
\( s = \) spacing in direction 1 closest to the tensile extreme fibers (in.).
\( s_2 = \) spacing in the perpendicular direction (in.).
\( d_c = \) concrete cover to centroid of reinforcement (in.).
\( d_b = \) diameter of the reinforcement in direction 1 closest to the concrete outer fibers (in.).

The tolerable crack widths in concrete elements are given in Table 36.1. In SI units, Equation 36.58 therefore becomes:

\[
w_{\text{max}} (\text{mm}) = 0.14k\beta f_y \sqrt{G}
\]

where \( f_y \) is in megapascals, and \( s_1, s_2, d_c, \) and \( d_b \) are in millimeters.

### 36.3.8 Development of Reinforcement

#### 36.3.8.1 Development of Deformed Bars in Tension

The full development length \( (\ell_d) \) for deformed bars or wires is obtained by applying multipliers to a basic theoretical development length \( (\ell_{db}) \) in terms of the bar diameter \( (d_b) \) and other multipliers as follows:

\[
\ell_d = \frac{3}{40} \frac{f_y \psi_1 \psi_e \psi_s}{\left( \frac{c_l + K_s}{d_b} \right)}
\]  

(36.59)

The value of \( \sqrt{f_y'^2} \) should not exceed 100 psi \((\leq 6.9 \text{ MPa})\) in all computations.

#### 36.3.8.2 Modifying Multipliers of Development Length for Bars in Tension

- \( \psi_1 = \) bar location factor. For horizontal reinforcement, when more than 12 in. of fresh concrete is below the development length or splice (top reinforcement), \( \alpha = 1.3 \); for other reinforcement, \( \alpha = 1.0 \).
- \( \psi_e = \) coating factor. For epoxy-coated bars or wires with cover less than 3\( d_b \) or clear spacing less than 3\( d_b \), \( \beta = 1.5 \); for all other epoxy-coated bars or wires, \( \psi_e = 1.2 \); for uncoated reinforcement \( \psi_e = 1.0 \). However, the product \( \psi_1 \psi_e \) should not exceed 1.7.
- \( \psi_s = \) bar size factor. For No. 6 and smaller bars and deformed wires (No. 20 and smaller, SI), \( \gamma = 0.8 \); for No. 7 and larger bars (No. 25 and larger, SI), \( \gamma = 1.0 \).
- \( c = \) spacing or cover dimension (in.). Use the smaller of either the distance from the center of the bar to the nearest concrete surface or one half the center-to-center spacing of the bars being developed.
- \( K_s = \) transverse reinforcement index, which is equal to \((40A_{tr}/sn)\), where \( A_{tr} = \) total cross-sectional area of all transverse reinforcement within \( l_d \) that crosses the potential plane of splitting adjacent to the reinforcement being developed (in.\(^2\)). Also, \( f_y = 60,000 \) psi is the strength value used in the development of the \( A_{tr} \) expression; \( s \) is the maximum spacing of transverse reinforcement within \( l_d \) center-to-center (in.) (mm); and \( n \) is the number of bars or wires being developed along the plane of splitting. The ACI 318 Code permits using \( K_s \) as a conservative design simplification even if transverse reinforcement is present.
- \( \lambda = \) lightweight-aggregate concrete factor. When lightweight aggregate concrete is used \( \lambda = 0.75 \); however, when \( f_y \) is specified, use \( \lambda = 6.7 \sqrt{f_y'/f_y} \). For all other concrete, \( \lambda = 1.0 \). The minimum development length in all cases is 12 in.
- \( \lambda_r = \) excess reinforcement factor. ACI 318 permits the reduction of \( \ell_d \) if the longitudinal flexural reinforcement is in excess of that required by analysis except where anchorage or development for \( f_y \) is specifically required or the reinforcement is designed for seismic effects. The reduction multiplier \( \lambda_r = (A_i \text{ required})/(A_i \text{ provided}) \) and \( \lambda_{r2} = f_y/60,000 \) for cases where \( f_y > 60,000 \) psi. In lieu of using a refined computation for the development length of Equation 36.59, Table 36.4 can be utilized for typical construction practices by using a value of \( \psi_1 \) and \( \psi_e = 1.0 \) and \( f_y' = 4000 \) psi.
Table 36.5 gives minimum development length $l_d$ (in.) in lieu of calculations using Table 36.4. In these two tables, the following assumptions are made: (1) The side cover is 1.5 in. on each side; (2) No. 3 stirrups are used for bars No. 11 or smaller; (3) No. 4 stirrups are used for bars No. 14 or No. 18; and (4) stirrups are bent around four bar diameters, so the distance from the centroid of the bar nearest the side face of the beam to the inside face of the No. 3 stirrup is taken as 0.75 in. for bars No. 11 or smaller and is equal to the longitudinal bar radius for No. 14 and No. 18 bars.

### Development of Deformed Bars in Compression and the Modifying Multipliers

Bars in compression require shorter development length than bars in tension. This is due to the absence of the weakening effect of the tensile cracks; hence, the expression for the basic development length is:

$$l_{d_k} = 0.02 \frac{d_k f_y}{\lambda_c \sqrt{f'c}}$$  \hspace{1cm} (36.60a)

$$l_{d_k} = 0.0003d_k f_y$$  \hspace{1cm} (36.60b)

with the modifying multiplier for (1) excess reinforcement, $\lambda_c = (A_r \text{ required})/(A_r \text{ provided})$; and (2) spirally enclosed reinforcement, $\lambda_{s1} = 0.75$. 

---

**TABLE 36.4a** Simplified Development Length $l_d$ Equations

<table>
<thead>
<tr>
<th>Clear spacing of bars being developed or spliced not less than $d_b$, clear cover not less than $d_c$, and stirrups or ties throughout $l_d$ not less than the Code minimum</th>
<th>#6 and Smaller Bars and Deformed Wires</th>
<th>#7 and Larger Bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>when: $f'c = 4000$ psi $\psi_1, \psi_2, \lambda_c, \lambda_v \gamma = 1.0$ $\psi_s = 0.8$ $l_d = 38d_b$</td>
<td>$l_d = \frac{f'c}{25\sqrt{f'c}}$</td>
<td>$l_d = \frac{f'c}{20\sqrt{f'c}}$</td>
</tr>
<tr>
<td>when: $f'c = 4000$ psi $\psi_1, \psi_2, \lambda_c, \lambda_v \gamma = 1.0$ $\psi_s = 0.8$ $l_d = 38d_b$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Other cases | $l_d = 57d_b$ | $l_d = 72d_b$

Note: This is a general table for usual construction conditions giving the required development length for deformed bars of sizes No. 3 to No. 18.


**TABLE 36.4b** SI Development Length Simplified Expressions

<table>
<thead>
<tr>
<th>$\leq$ No. 20</th>
<th>$\geq$ No. 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_d = \frac{f'c}{2\sqrt{f'c}}$</td>
<td>$l_d = \frac{5f'c}{8\sqrt{f'c}}$</td>
</tr>
<tr>
<td>$l_d = \frac{3f'c}{4\sqrt{f'c}}$</td>
<td>$l_d = \frac{15f'c}{16\sqrt{f'c}}$</td>
</tr>
</tbody>
</table>
36.3.8.4 Development of Bundled Bars in Tension and Compression

If bundled bars are used in tension or compression, \( l_d \) has to be increased by 20% for three-bar bundles and 33% for four-bar bundles, and \( f' \) should not be taken as greater than 100 psi. A unit of bundled bars is treated as a single bar of a diameter derived from the equivalent total area for the purpose of determining the modifying factors. Although the splice and development lengths of bundled bars are based on the diameter of individual bars increased by 20 or 33% as applicable, it is necessary to use an equivalent diameter of the entire bundle derived from the equivalent total area of bars when determining the factors that consider cover and clear spacing and represent the tendency of concrete to split.

36.3.8.5 SI/Metric Conversion

Where \( f_{ys} \) is in megapascals, Equation 36.59 becomes:

\[
\frac{15f_{ys} \psi \psi \psi}{16 \sqrt{f'} \left( \frac{c_{2} + K_{n}}{d_{b}} \right)} \text{ and } K_{n} = \frac{1.6A_{n}}{mn}
\]

36.3.8.6 Development of Welded Deformed Wire Fabric in Tension

The development length \( (l_{d}) \) for deformed welded wire fabric should be taken as the \( l_{d} \) value obtained from Equation 36.59 or Table 36.4 multiplied by a fabric factor. The fabric factor, with at least one cross wire within the development length and not less than 2 in. from the point of the critical section, should be taken as the greater of the following two expressions:

\[
\text{TABLE 36.5 Tension Reinforcement and Development Length}
\]

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Cross-Sectional Area (in.²)</th>
<th>Bar Diameter (in.)</th>
<th>Development Length ((l_{d})^{a,b}) (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(s \geq 2d_{b} ) or ( d_{b} )</td>
<td>Clear Cover ( \geq d_{b} )</td>
</tr>
<tr>
<td>3</td>
<td>0.11</td>
<td>0.375</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>0.20</td>
<td>0.500</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>0.31</td>
<td>0.625</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>0.44</td>
<td>0.750</td>
<td>29</td>
</tr>
<tr>
<td>7</td>
<td>0.60</td>
<td>0.875</td>
<td>42</td>
</tr>
<tr>
<td>8</td>
<td>0.79</td>
<td>1.000</td>
<td>48</td>
</tr>
<tr>
<td>9</td>
<td>1.00</td>
<td>1.128</td>
<td>54</td>
</tr>
<tr>
<td>10</td>
<td>1.27</td>
<td>1.270</td>
<td>61</td>
</tr>
<tr>
<td>11</td>
<td>1.56</td>
<td>1.410</td>
<td>68</td>
</tr>
<tr>
<td>14</td>
<td>2.25</td>
<td>1.693</td>
<td>82</td>
</tr>
<tr>
<td>18</td>
<td>4.00</td>
<td>2.257</td>
<td>108</td>
</tr>
</tbody>
</table>

\(a\) For compression development length, \( l_{d} \) multiplier \( \times l_{db} \).

\(b\) Multiply table values by \( \alpha = 1.3 \) for top reinforcement, \( \lambda = 0.75 \) for lightweight aggregate, \( \psi_{e} = 1.5 \) for epoxy-coated bars with cover less than 3\(d_{b}\) or clear spacing less than 6\(d_{b}\), and \( \beta = 1.2 \) for other epoxy-coated bars. Minimum \( l_{d} \) for all cases is 12 in.

Note: For \( f' = 4000 \text{ psi normal weight concrete}, f_{ys} = 60,000 \text{ psi steel (}\psi_{e}, \psi_{s}, \lambda = 1.0; \gamma = 0.8 \text{ for #6 bars or smaller and 1.0 for #7 bars and larger}). For \( f' \) values different from 4000 psi, multiply table values by \( \frac{4000}{f'} \). For \( f_{ys} = 40,000 \text{ psi, multiply by 2/3. } f' \) should not exceed 100.

but should not be taken as greater than 1.0. Here, \( s \) is the spacing of wire to be developed or spliced (in.).

### 36.4 Prestressed Concrete

#### 36.4.1 General Principles

Reinforced concrete is weak in tension but strong in compression. To maximize the utilization of its material properties, an internal compressive force is induced on the structural element through the use of highly stressed prestressing tendons to precompress the member prior to application of the external gravity live load and superimposed dead load. A typical effect of the prestressing action shown in Figure 36.14 uses a straight tendon, as is usually the case for precast elements (Nawy, 2006). For cast-in-place elements, the tendon can be either harped, or, as is usually the case, it can be draped in a parabolic form. Figure 36.15 illustrates the stress and strain distributions across the beam depth and the forces acting on the section in a prestressed concrete beam and the compressive stress block of the section.

**Stresses due to initial prestressing plus self-weight:**

\[
\begin{align*}
\sigma' &= \frac{P_t}{A_t} \left(1 - \frac{e_{ct}}{r_t^2}\right) \frac{M_D}{S_t} \\
\sigma_b &= -\frac{P_t}{A_t} \left(1 + \frac{e_{ct}}{r_t^2}\right) + \frac{M_D}{S_b}
\end{align*}
\]

#### 36.4.2 Stress and Strain Distribution

**Stresses at service load:**

\[
\begin{align*}
\sigma' &= -\frac{P_t}{A_t} \left(1 - \frac{e_{ct}}{r_t^2}\right) \frac{M_T}{S_t} \\
\sigma_b &= -\frac{P_t}{A_t} \left(1 + \frac{e_{ct}}{r_t^2}\right) + \frac{M_T}{S_b}
\end{align*}
\]

© 2008 by Taylor & Francis Group, LLC
FIGURE 36.15 Stress and strain distribution across prestressed concrete beam depth: (a) beam cross-section; (b) strain across depth; (c) actual stress block; and (d) assumed equivalent block. (From Nawy, E.G., Prestressed Concrete: A Fundamental Approach, 5th ed., Prentice Hall, Upper Saddle River, NJ, 2006.)
36.4.2 Minimum Section Modulus for Variable Tendon Eccentricity

\[ S' \geq \frac{(1-\gamma)M_D + M_{sd} + M_L}{\gamma f_u - f_t} \]  

(36.65a)

\[ S_b \geq \frac{(1-\gamma)M_D + M_{sd} + M_L}{\gamma f'_t - f'_u} \]  

(36.65b)

where:

\( \gamma \) = percentage loss in prestress.
\( M_D \) = self-weight moment.
\( M_{sd} \) = superimposed dead load moment.
\( M_L \) = live load moment.
\( f_u \) = initial tensile stress in concrete.
\( f_c \) = service load concrete compressive strength.
\( f_t \) = service load concrete tensile strength.
\( f_{ti} \) = initial compressive stress in concrete.
\( S' \) = section modulus at top fibers (simple span).
\( S_b \) = section modulus at bottom fibers (simple span).

36.4.3 Minimum Section Modulus for Constant Tendon Eccentricity

\[ S' \geq \frac{M_D + M_{sd} + M_L}{\gamma f_u - f_t} \]  

(36.66a)

\[ S^b \geq \frac{M_D + M_{sd} + M_L}{f_t - \gamma f'_u} \]  

(36.66b)

36.4.4 Maximum Allowable Stresses

36.4.4.1 ACI 318 Code Concrete Stresses

\[ f'c = 0.75 f'_s \text{ psi} \]
\[ f_u = 0.60 f'_c \text{ psi} \]

\( f_u = \sqrt{f'_c} \text{ psi on span } (\sqrt{f'_c} / 2 \text{ MPa}) = \sqrt{f'_c} \text{ psi on support } (\sqrt{f'_c} / 2 \text{ MPa}) \)

\( f_t = 0.45f'_s \text{ or } 0.60f'_s \text{ where permitted by ACI 318.} \)

\( f_t = \sqrt{f'_t} \text{ psi } (\sqrt{f'_t} / 2 \text{ MPa}) = 12\sqrt{f'_c} \text{ psi if deflection is verified } (\sqrt{f'_c} / 2 \text{ MPa}) \)

36.4.4.2 Reinforcing Tendon Stresses

Tendon jacking:

\[ f_{pu} = 0.94f_{py} \leq 0.80f_{pu} \]

Immediately after prestress transfer:

\[ f_{pu} = 0.82f_{pu} \leq 0.74f_{pu} \]
Post-tensioned members at anchorage immediately after tendon anchorage:

\[ f_p = 0.70f_{pu} \]

where:
- \( f_p \) = ultimate design stress allowed in tendon.
- \( f_y \) = yield strength of tendon.
- \( f_{pu} \) = ultimate strength of tendon.

A prestressed concrete section is designed for both the service load and the ultimate load. A typical distribution of stress at service load at midspan is shown in Figure 36.15. Expressions for the ultimate load evaluation are essentially similar to those of reinforced-concrete elements, taking into consideration that both prestressing tendons and mild steel bars are used. Note the similarity between Figure 36.15 and Figure 36.7. For extensive design and analysis details, refer to Nawy (2006).

### 36.5 Shear and Torsion in Prestressed Elements

#### 36.5.1 Shear Strength: ACI Short Method When \( f_{pc} > 0.40f_{pu} \)

The nominal shear stress of the concrete in the web is:

\[ V_i (lb) = \left( 0.60 \lambda \sqrt{f'_c} + \frac{700V_d}{\lambda} \right) b_v d \]  
\[ V_i \geq 2\lambda \sqrt{f'_c} b_v d \leq 5\lambda \sqrt{f'_c} b_v d, \quad \frac{V_d}{M_u} \leq 1.0 \]  
\[ V_i (Newton) = \left( \frac{\lambda \sqrt{f'_c}}{20} + \frac{V_d}{M_u} \right) b_v d \]  
\[ V_i \geq \frac{\lambda \sqrt{f'_c}}{6} b_v d \leq 0.40\lambda \sqrt{f'_c} b_v d, \quad \frac{V_d}{M_u} \leq 1.0 \]

#### 36.5.2 Detailed Method

The smaller of the two values obtained from flexural shear \( (V_i) \) or web shear \( (V_{cw}) \) in the following expressions has to be used in the design of the web reinforcement in prestressed concrete members.

#### 36.5.2.1 Flexural Shear

\[ V_{ci} (lb) = 0.60\lambda \sqrt{f'_c} b_v d + V_d + \frac{V_d M_u}{M_{max}} \geq 1.7\lambda \sqrt{f'_c} b_v d \]  

where:
- \( V_{ci} \) = flexural shear force.
- \( M_{ci} = \lambda b_v \left( \lambda \sqrt{f'_c} + f_{ce} - f_d \right) \).
- \( S_b \) = section modulus at the extreme tensile fibers.
- \( V_d \) = shear force at section due to unfactored dead load.
- \( V_i \) = factored shear force due to externally applied load.
- \( f_{ce} \) = compressive stress in concrete due to effective prestress only at the tension face of the section.
- \( f_d \) = stress due to unfactored dead load at extreme fibers in tension.
Proportioning Concrete Structural Elements by the ACI 318-08 Code

36.5.2.2 Web Shear

\[
V_{cw} = \text{web shear force}
\]

\[
V_{cw} (\text{lb}) = \left(3.5\lambda \sqrt{f_c'} + f_y\right)b_wd + V_p
\]

\[
V_{cw} (\text{Newton}) = \left(0.3\lambda \sqrt{f_c'} + f_y\right)b_wd + V_p
\]

where:

\(f_c'\) = compressive stress at center of gravity of section due to externally applied load.

\(V_p\) = vertical component of prestressing force.

The critical section for calculating \(V_u\) and \(T_u\) is taken at distance \((h/2)\) from the face of the support.

36.5.3 Minimum Shear Reinforcement

For prestressed members subjected to shear, the minimum transverse web stirrups are the smaller of:

\[
A_s (\text{in.}^2) = \frac{50b_w s}{f_y}
\]

or

\[
A_s (\text{in.}^2) = \frac{A_p d_p}{80f_y d} \sqrt{\frac{d_p}{b_w}}
\]

where \(f_y\) is in psi and \(s\) is the web reinforcement spacing.

36.5.4 Torsional Strength

As discussed earlier, the nominal torsional strength \((T_u)\) is disregarded, and all of the torque is assumed by longitudinal bars and the transverse closed hoops. The expressions used in the case of prestressed concrete elements are essentially the same as those for reinforced-concrete elements with the following adjustments for Equation 36.35 and Equation 36.36. Multiply the right side by:

\[
\sqrt{1 + \frac{3f_p}{f_c'}}
\]

For hollow sections, the left side of Equation 36.37 becomes:

\[
\left(\frac{V_u}{b_wd}\right) + \left(\frac{T_p b_w}{1.7A_{sh}^2}\right)
\]

The maximum spacing of the closed hoops is \(1/8p_h \leq 12\) in., and the longitudinal bar diameter is not less than \(1/16\), where \(s\) is the spacing of the hoop steel.
36.6 Walls and Footings

The design of walls and footings should be viewed in the context of designing a one-way or two-way cantilever slab in the case of footings and one-way vertical cantilevers in the case of reinforced-concrete walls. The criteria and expressions for proportioning their geometry are the same as those presented in earlier sections of this chapter. Shear $V_u$ in one-way footings is taken at a distance $d$ from the face of the vertical concrete wall or columns and at $d/2$ in the case of two-way footings. The nominal shear strength (capacity) $V_c$ of the one-way slab footing is:

$$V_c = 2\lambda \sqrt{f'_c b_o d}$$

(36.71)

For two-way slab footings, the nominal shear strength $V_c$ should be the smallest of:

$$V_c = 4\lambda \sqrt{f'_c b_o d}$$

(36.72a)

or

$$V_c = \left(2 + \frac{4}{\beta_c}\right) \lambda \sqrt{f'_c b_o d}$$

(36.72b)

or

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right) \lambda \sqrt{f'_c b_o d}$$

(36.72c)

where $b_o$ is the perimeter shear failure length at distance $d/2$ from all faces of columns. If the column size is $c_1 \times c_2$, then:

- $b_o = 2(c_1 + d/2) + 2(c_2 + d/2)$ for an interior column.
- $\beta_c$ = ratio of long side/short side of reaction area.
- $\alpha_s$ = 40 for interior columns, 30 for end columns, and 20 for corner columns.

The same requirement for shear in Equation 36.72 applies to the shear design of two-way action structural slabs and plates.

Acknowledgments

This chapter is based on material appearing in the previous edition of this Handbook; from Fundamentals of High-Performance Concrete, 2nd ed., by E.G. Nawy (John Wiley & Sons, 2001); from Reinforced Concrete: A Fundamental Approach, 6th ed., by E.G. Nawy (Prentice Hall, 2008); from Prestressed Concrete: A Fundamental Approach, 5th ed., by E.G. Nawy (Prentice Hall, 2006); and from various committee reports and standards of the American Concrete Institute, Farmington Hills, MI.

References

ACI Committee 224. 2001. Control of Cracking in Concrete Structures, ACI 244R. American Concrete Institute, Farmington Hills, MI.
ACI Committee 350. 2006. Code Requirements for Environmental Engineering Concrete Structures and Commentary, ACI 350 ERTA. American Concrete Institute, Farmington Hills, MI.