4.1 Creep and Shrinkage Deformations in Concrete ..........4-1

Creep, or lateral material flow, is the increase in strain with time due to sustained load. Initial deformation due to load is the elastic strain, while the additional strain or time-dependent deformation due to the same sustained load is the creep strain. Drying shrinkage, on the other hand, is the decrease in volume
of the concrete element when it loses moisture due to evaporation. Because every infrastructure element under load will suffer this long-term deformation that has to be accounted for in a durable system, this aspect has been chosen as one of the major topics for consideration in this chapter.

If we look at the three-dimensional representation in Figure 4.1 and its two-dimensional counterpart in Figure 4.2, we can see that the nonlinear relationship in the creep and shrinkage behavior makes it difficult to come up with an exact model of prediction. This has been a challenge for researchers from the 1920s onwards and is why there are so many models of prediction (ACI 209, 1992; Bazant and Boweja, 1995a,b, 2000a,b; Bazant and Murphy, 1995; Branson, 1971, 1977; CEB-FIP, 1990; Gardner, 2000; Gardner and Lockman, 2001; Ross, 1937). Because of the incomplete reversibility of both creep and shrinkage strains, we can observe cracking, sagging of elements, and progressive deterioration as the strain continues to increase with time. Figure 4.2a shows a section of the three-dimensional model presented in Figure 4.1 parallel to the plane that contains the stress and deformation axes at time $t_0$. The figure indicates that both elastic and creep strains are linearly proportional to applied stress. In a similar manner, Figure 4.2b illustrates a section parallel to the plane that contains the time and strain axes at a stress $f_1$; hence, it shows the familiar creep time and shrinkage time relationships.

The current ACI 318 code expressions due to Branson are still the acceptable general-purpose expressions for the prediction of creep and shrinkage as embodied in the ACI 209 model, and the designer can use other methods such as the CEB-FIP (1990), Bazant’s B3 model (Bazant and Boweja, 1995a,b, 2000a,b), or Gardner’s GL 2000 model (Gardner, 2000; Gardner and Lockman, 2001) for refined predictions (Nawy, 2006a,b, 2008). Recent work by Nassif at Rutgers (Saksawang et al., 2005), shown in Figure 4.3, indicates that the ACI 209 model seems to be a close best fit for creep prediction in high-strength, high-performance concrete; Figure 4.4 shows essentially similar characteristics for shrinkage but with lesser correlation.

### 4.2 Creep Deformations in Concrete

Creep or lateral material flow is the increase in strain with time due to sustained load. Initial deformation due to load is the *elastic strain*, while the additional strain due to the same sustained load is the *creep strain*. This practical assumption is quite acceptable, as the initial recorded deformation includes few time-dependent effects. Figure 4.5 illustrates the increase in creep strain with time; in the case of
Long-Term Effects and Serviceability

It can be seen that the rate of creep decreases with time. Creep cannot be measured directly but is determined only by deducting elastic strain and shrinkage strain from the total deformation. Although shrinkage and creep are not independent phenomena, it can be assumed that superposition of strains is valid; hence,

\[
\varepsilon_{\text{total}} = \varepsilon_{\text{elastic}} + \varepsilon_{\text{creep}} + \varepsilon_{\text{shrinkage}}
\]

An example of the relative numerical values of strain due to elastic strain creep and to shrinkage is presented in the figure for a normal concrete specimen subjected to 900 psi in compression (Nawy, 2001; Ross, 1937). These relative values illustrate that stress–strain relationships for short-term loading in...
Normally reinforced or plain concrete elements lose their significance and the effects of long-term loadings become dominant on the behavior of a structure. In cases of large heavily reinforced columns in buildings, elastic strain can be a more significant component of the total strain.

Numerous tests have indicated that creep deformation is proportional to the applied stress, but the proportionality is valid only for low stress levels. The upper limit of the relationship cannot be determined accurately but can vary between 0.2 and 0.5 of the ultimate strength $f'$. This range in the limit of the proportionality is expected due to the large number of microcracks that exist at about 40% of the ultimate load.

As in the case of shrinkage, creep is not completely reversible. If a specimen is unloaded after a period of being under a sustained load, an immediate elastic recovery is obtained that is less than the strain precipitated on loading. The instantaneous recovery is followed by a gradual decrease in strain, called creep recovery. The extent of the recovery depends on the age of the concrete when loaded; older concretes present higher creep recoveries, while residual strains or deformations become frozen in the structural element, as shown in Figure 4.6.
4.2.1 Creep Effects

As in shrinkage, creep increases the deflection of beams and slabs and causes loss of prestress in prestressed concrete elements. In addition, the initial eccentricity of a reinforced concrete column increases with time due to creep, resulting in the transfer of the compressive load from the concrete to the steel in the concrete section. When the steel yields, additional load has to be carried by the concrete; consequently, the resisting capacity of the column is reduced, and the curvature of the column increases further, resulting in overstress in the concrete and leading to failure. Similar behavior occurs in axially loaded columns.

4.2.2 Rheological Models

Rheological models are mechanical devices that portray the general deformation behavior and flow of materials under stress. A model is basically composed of elastic springs and ideal dashpots denoting stress, elastic strain, delayed elastic strain, irreversible strain, and time. The springs represent the proportionality between stress and strain, and the dashpots represent the proportionality of stress to the rate of strain. A spring and a dashpot in parallel form a Kelvin unit, and in series they form a Maxwell unit.

Two rheological models are discussed here: the Burgers model and the Ross model. The Burgers model (Figure 4.7) is shown because it can approximately simulate the stress–strain–time behavior of concrete at the limit of proportionality, with some limitations. This model simulates the instantaneous recoverable strain (a), the delayed recoverable elastic strain in the spring (b), and the irreversible time-dependent strain in dashpots (c and d). The weakness in this model is that it continues to deform at a uniform rate as long as the load is sustained by the Maxwell dashpot—a behavior not similar to concrete, where creep reaches a limiting value with time, as shown in Figure 4.7. A modification in the form of the Ross rheological model (Ross, 1958) (Figure 4.8) can eliminate this deficiency. In this model, A represents the Hookian direct proportionality of the stress-to-strain element, D represents the Newtonian element, and B and C are the elastic springs that can transmit the applied load \( P(t) \) to the enclosing cylinder walls by direct friction. Because each coil has a defined frictional resistance, only those coils whose resistance equals the applied load \( P(t) \) are displaced; the others remain unstressed, symbolizing irreversible deformation in concrete. As the load continues to increase, it overcomes the spring resistance of unit B,
pulling the spring from the dashpot and signifying failure in a concrete element. More rigorous models have been used, such as Roll’s model, to assist in predicting the creep strains. Mathematical expressions for such predictions can be very rigorous. One convenient expression from Ross defines creep \( \left( C \right) \) under load after time interval \( t \) as follows:

\[
C = \frac{t}{a+bt}
\]  

where \( a \) and \( b \) are constants that can be determined from tests. As will be discussed in the next section, this model seems to represent the creep deformation of concrete and is the background for the ACI code equations for creep.

### 4.3 Creep Prediction

#### 4.3.1 Creep Prediction for Standard Conditions

Creep and shrinkage are interrelated phenomena because of the similarity of the variables affecting both, including the forms of their strain–time curves, as seen in Figure 4.2. The ACI (ACI Committee 209, 1992) proposed a similar model for expressing both creep and shrinkage behavior. The expression for creep is as follows:

\[
C_i = \frac{t^\alpha}{a+t^\alpha} C_u
\]  

where \( a \) and \( \alpha \) are experimental constants and \( t \), in days, is the duration of loading.

Work by Branson (1971, 1977) formed the basis for Equation 4.2 and Equation 4.3 in a simplified creep evaluation. The additional strain \( (\varepsilon_{\text{cu}}) \) due to creep can be defined as:

\[
\varepsilon_{\text{cu}} = \rho_u f_u
\]  

where:

- \( \rho_u \) = unit creep coefficient, generally referred to as specific creep.
- \( f_u \) = stress intensity in the structural member corresponding to initial unit strain \( \varepsilon_{\text{cu}} \).
4.3.2 Factors Affecting Creep

Creep is greatly affected by concrete constituents. The coarse-aggregate modulus affects the creep strain level, but the cementitious paste and its shear–friction interaction with the aggregate are constituents that significantly influence the time-dependent, load-induced strain. Other factors are environmental effects. A summary of these factors follows:

\[ C_u = \frac{e_u}{e_i} = \rho_t E_c \]  

if \( C_u \) is the ultimate creep coefficient. An average value of \( C_u \) is 2.35.

Branson’s model, verified by extensive tests, relates the creep coefficient \( (C_t) \) at any time to the ultimate creep coefficient for standard conditions as follows:

\[ C_t = \frac{t^{0.6}}{10 + t^{0.6}} C_u \]  

or, alternatively,

\[ \rho_t = \frac{t^{0.6}}{10 + t^{0.6}} \]  

where \( t \) is the time in days during which the load is applied. Standard conditions are summarized in Table 4.1 for both creep and shrinkage (ACI Committee 209, 1992).

### TABLE 4.1 Standard Conditions for Creep and Shrinkage Factors

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Factors</th>
<th>Variable Considered</th>
<th>Standard Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete composition</td>
<td>Cement paste content</td>
<td>Type of cement</td>
<td>Type I and Type III</td>
</tr>
<tr>
<td></td>
<td>Water/cement ratio</td>
<td>Slump</td>
<td>2.7 in. (70 mm)</td>
</tr>
<tr>
<td></td>
<td>Mixture proportions</td>
<td>Air content</td>
<td>≤6%</td>
</tr>
<tr>
<td></td>
<td>Aggregate characteristics</td>
<td>Fine aggregate percentage</td>
<td>50%</td>
</tr>
<tr>
<td></td>
<td>Degree of compaction</td>
<td>Cement content</td>
<td>470 to 752 lb/yd³ (279 to 446 kg/m³)</td>
</tr>
<tr>
<td>Initial curing</td>
<td>Length of initial curing</td>
<td>Moist-cured</td>
<td>7 days</td>
</tr>
<tr>
<td></td>
<td>Curing temperature</td>
<td>Moist-cured</td>
<td>73.4 ± 4°F (23 ± 2°C)</td>
</tr>
<tr>
<td></td>
<td>Curing humidity</td>
<td>Relative humidity</td>
<td>≥95</td>
</tr>
<tr>
<td>Member geometry and environment</td>
<td>Concrete temperature</td>
<td>Concrete temperature</td>
<td>73.4 ± 4°F (23 ± 2°C)</td>
</tr>
<tr>
<td>Geometry</td>
<td>Concrete water content</td>
<td>Ambient relative humidity</td>
<td>40%</td>
</tr>
<tr>
<td></td>
<td>Size and shape</td>
<td>Volume/surface ratio (V/S)</td>
<td>V/S = 1.5 in. (38 mm)</td>
</tr>
<tr>
<td></td>
<td>Minimum thickness</td>
<td>6 in. (150 mm)</td>
<td></td>
</tr>
<tr>
<td>Loading history</td>
<td>Concrete age at load</td>
<td>Moist-cured</td>
<td>7 days</td>
</tr>
<tr>
<td>Loading (only creep)</td>
<td>Application</td>
<td>Steam-cured</td>
<td>1–3 days</td>
</tr>
<tr>
<td></td>
<td>Duration</td>
<td>Sustained load</td>
<td>Sustained load</td>
</tr>
<tr>
<td></td>
<td>Duration of unloading period</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>Number of unloading cycles</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Stress conditions</td>
<td>Type of stress and distribution across the section</td>
<td>Compressive stress</td>
<td>Axial compression</td>
</tr>
<tr>
<td>Stress/strength ratio</td>
<td>Stress/strength ratio</td>
<td>≤0.50</td>
<td></td>
</tr>
</tbody>
</table>

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• **Sustained load.** Creep as a result of sustained load is proportional to the sustained stress and is recoverable up to 30 to 50% of the ultimate strain.

• **Water/cementitious materials ratio** (w/(c + p) or w/cm). The higher this ratio, the larger the creep, as seen in Figure 4.9, which relates specific creep to the w/(c + p) ratio (Mindess et al., 1999).

• **Aggregate modulus and aggregate/paste ratio.** For constant paste-volume content, an increase in aggregate volume decreases creep. As an example, an increase from 65 to 75% lowered creep by 10%. This behavior is the same whether the coarse aggregate is natural stone or lightweight artificial aggregate.

• **Age at time of loading.** The older the concrete at the time of loading, the smaller the induced creep strain for the same load level.

• **Relative humidity.** Reconditioning the concrete at a lower relative humidity before applying the sustained external load reduces the resulting creep strain. If creep is considered in two categories—drying creep and wetting creep—the creep strain develops irrespective of the direction of change (Mindess et al., 1999), provided the exposure is above 40%.

• **Temperature.** Creep increases with increase in temperature if the concrete is maintained at elevated temperatures while under sustained load. It increases in a linear manner up to a temperature of 175°F (80°C). Its value at this temperature level is almost three times the creep value at ambient temperatures.

• **Concrete member size.** Creep strain decreases with increasing thickness of the concrete member.

• **Reinforcement.** Creep effects are reduced by use of reinforcement in the compressive zones of concrete members.

### 4.3.3 Creep Prediction for Nonstandard Conditions

As the standard conditions for creep described in Table 4.1 change, corrective modifying multipliers have to be applied to the ultimate creep coefficient \( C_u \) in Equation 4.5. If the average ultimate creep \( C_u \) is 2.35 for standard conditions, it has to be adjusted by a multiplier \( \gamma_{C_{u-R}} \) so:

\[
C_u = 2.35 \gamma_{C_{u-R}} \tag{4.7a}
\]

Here, \( \gamma_{C_{u-R}} \) has component coefficients that account for the change in conditions enumerated in the preceding section. ACI Committee 209 recommends, in detailed tabular form, the various component conditions.

![FIGURE 4.9 Water/cement ratio effect on the relative specific creep. (From Mindess, S. et al., Concrete, Prentice Hall, Upper Saddle River, NJ, 1999. With permission.)](image)
coefficients for the $\gamma_{CR}$ multiplier (ACI Committee 209, 1992). These are generally based on Branson's (1977) studies. Tabulated values are given in graphical form (ACI Committee 435, 1995; Branson, 1977; Meyers and Thomas, 1983) in Figure 4.10 for the multiplier as follows:

$$\gamma_{CR} = K_h'K_c'K_s'K_{ac}K_{cr}$$ (4.7b)

where:

- $\gamma_{CR} = 1$ for standard conditions.
- $K_h'$ = relative humidity factor.
- $K_c'$ = minimum member thickness factor.
- $K_s'$ = concrete consistency factor.
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$K_f = \text{fine aggregate content factor.}$

$K_{ac} = \text{air-content factor.}$

$K_{ao} = \text{age of concrete at load application factor.}$

### 4.3.4 CEB-FIP Model Code for Creep

The total stress-dependent strain per unit stress in the CEB-FIP 90-99 model is given by the following expression:

$$\varepsilon_{cr(t,t_0)} = \frac{1}{E_{t_0}} \frac{\Phi_{(t,t_0)}}{E_{28}}$$  \hspace{1cm} (4.8)

where:

- $E_{t_0}$ = modulus of elasticity at age of loading (MPa).
- $E_{28}$ = modulus of elasticity at 28 days (MPa).
- $\Phi_{(t,t_0)}$ = creep coefficient dependent on several factors, each defined by a separate equation relating to cement type, relative humidity, concrete strength, and age of concrete.

### 4.4 Shrinkage in Concrete

#### 4.4.1 General Shrinkage Behavior

In general, the two types of shrinkage are plastic shrinkage and drying shrinkage; carbonation shrinkage is another form of shrinkage. Plastic shrinkage occurs during the first few hours after placing fresh concrete in forms. Exposed surfaces such as floor slabs are more easily affected by exposure to dry air because of their large contact surface. In such cases, moisture evaporates from the concrete surface faster than it is replaced by bleed water from the lower layers. Drying shrinkage, on the other hand, occurs after the concrete has already attained its final set and a good portion of the chemical hydration process in the cement gel has been accomplished. Drying shrinkage is the decrease in the volume of a concrete element when it loses moisture by evaporation. The opposite phenomenon—volume increase through water absorption—is termed swelling. In other words, shrinkage and swelling represent water movement out of or into the gel structure of a concrete specimen that is caused by the difference in humidity or saturation levels between the specimen and the surroundings, regardless of the external load.

Shrinkage is not a completely reversible process. If a concrete unit is saturated with water after having fully shrunk, it will not expand to its original volume. Figure 4.11 relates the increase in shrinkage strain

![FIGURE 4.11 Concrete shrinkage vs. time curve. (From Nawy, E.G., Reinforced Concrete: A Fundamental Approach, 6th ed., Prentice Hall, Upper Saddle River, NJ, 2008.)](image-url)
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\[(\varepsilon_{sh})\] with time. The rate decreases with time, as older concretes are more resistant to environmental effects and consequently undergo less shrinkage, so the shrinkage strain becomes almost asymptotic with time. Several factors affect the magnitude of drying shrinkage:

- **Aggregate.** Aggregate acts to restrain the shrinkage of cement paste; hence, concretes with a high aggregate content are less vulnerable to shrinkage. In addition, the degree of restraint of a given concrete is determined by the properties of the aggregates; those with a high modulus of elasticity or with rough surfaces are more resistant to the shrinkage process (see Figure 4.12 and Figure 4.13) (Mindess et al., 1999).

- **Water/cementitious materials ratio.** Traditionally, it is accepted that the higher the water/cementitious ratio, the higher the shrinkage effects. Figure 4.14 is a typical plot relating shrinkage to aggregate content and, significantly, to the water/cement ratio. More recent work (Suprenant and Malisch, 2000), shown in Figure 4.15, indicates only a slight increase in average drying shrinkage with increase in water content.

- **Size of the concrete element.** Both the rate and total magnitude of shrinkage decrease with an increase in the volume of the concrete element; however, the duration of shrinkage is longer for larger members as more time is needed for drying to reach the internal regions. It is possible that 1 year may be required for the drying process to begin at a depth of 10 inches from the exposed surface and 10 years for drying to begin at 24 inches below the external surface; large members may never dry out completely.

- **Ambient conditions of the medium.** The relative humidity of the medium greatly affects the magnitude of shrinkage; the rate of shrinkage is lower at high relative humidity. Temperature is another important factor, in that shrinkage becomes stabilized at low temperatures.

- **Amount of reinforcement.** Reinforced concrete shrinks less than plain concrete; the relative difference is a function of the reinforcement percentage.

- **Admixtures.** This effect varies depending on the type of admixture. An accelerator such as calcium chloride, used to accelerate the hardening and setting of the concrete, increases the shrinkage. Pozzolans can also increase the drying shrinkage, whereas air-entraining agents have little effect.

- **Type of cement.** Rapid-hardening cement shrinks somewhat more than other types, whereas shrinkage-compensating cements minimize or eliminate shrinkage cracking when used with restraining reinforcement.

---

**FIGURE 4.12** Aggregate modulus effect on shrinkage strain. (From Mindess, S. et al., *Concrete*, Prentice Hall, Upper Saddle River, NJ, 1999. With permission.)

<table>
<thead>
<tr>
<th>Modulus of Elasticity (GPa)</th>
<th>One-Year Shrinkage Strain × 10⁻⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>1</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>600</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

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Carbonation. Carbonation shrinkage is caused by the reaction between carbon dioxide (CO$_2$) present in the atmosphere and that present in the cement paste. The amount of combined shrinkage from carbonation and drying varies according to the sequence of carbonation and drying processes. If both phenomena take place simultaneously, less shrinkage occurs. The process of carbonation, however, is dramatically reduced at relative humidity below 50%.

4.4.2 Shrinkage Prediction for Standard Conditions

The mathematical model for shrinkage prediction in Equation 4.3 as an ACI 209 model is:

$$\left(\varepsilon_{SH}\right) = \frac{t^b}{b+t^b} \left(\varepsilon_{SH}\right)_u$$

(4.9)
where $\beta$ is a constant and $t$, in days, is the amount of time after curing that the concrete hardened. The value of the ultimate shrinkage strain at the standard conditions defined in Table 4.1 has the following range:

$\left(\varepsilon_{SH}\right)_u = 415\times10^{-6}$ to $1070\times10^{-6}$ in./in. (mm/mm)

stipulated by ACI Committee 209 (1992) is as follows:

Moist-cured for 7 days:

$\left(\varepsilon_{SH}\right)_u = 800\times10^{-6}$ in./in. (mm/mm)

Steam-cured for 1 to 3 days:

$\left(\varepsilon_{SH}\right)_u = 730\times10^{-6}$ in./in. (mm/mm)

A common sufficiently accurate average shrinkage strain in standard conditions for both moist-cured and steam-cured concretes (ACI Committee 209, 1992) is:

$\left(\varepsilon_{SH}\right)_u = 780\times10^{-6}$ in./in. (mm/mm)

The values of constant $b$ in the mathematical model of Equation 4.8 are $b = 35$ for 7-day moist-cured specimens and $b = 55$ for 1- to 3-day steam-cured specimens; hence, the shrinkage-strain prediction expressions for standard conditions become:

Shrinkage after 7 days of moist curing:

$\left(\varepsilon_{SH}\right)_t = \frac{t}{35+t}\left(\varepsilon_{SH}\right)_u$ \hspace{1cm} (4.10a)

where $t$ is the age of concrete in days after curing.

Shrinkage after 1 to 3 days of steam curing:

$\left(\varepsilon_{SH}\right)_t = \frac{t}{55+t}\left(\varepsilon_{SH}\right)_u$ \hspace{1cm} (4.10b)
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4.4.3 Shrinkage Prediction for Nonstandard Conditions

As the standard conditions for shrinkage described in Table 4.1 change, corrective modifying multipliers have to be applied to the ultimate value of the shrinkage strain ($\varepsilon_{SH}$) in Equations 4.10a and 4.10b. If $\gamma_{SH}$ is the shrinkage-adjusting multiplier, then the average ultimate shrinkage strain for nonstandard conditions becomes:

$$\varepsilon_{SH} = 780 \times 10^{-6} \gamma_{SH}$$  \hspace{1cm} (4.11a)
or

\[
(\varepsilon_{SH})_{u,n} = \gamma_{SH}(\varepsilon)_{u}
\]  
(4.11b)

where \((\varepsilon_{SH})_{u,n}\) is the average ultimate strain for nonstandard conditions; hence, for nonstandard conditions, Equations 4.10a and 4.10b, respectively, become:

\[
(\varepsilon_{SH})_{i} = \frac{1}{35+t}\gamma_{SH}(\varepsilon_{SH})_{u}
\]  
(4.12a)

and

\[
(\varepsilon_{SH})_{i} = \frac{1}{35+t}\gamma_{SH}(\varepsilon_{SH})_{u}
\]  
(4.12b)

The multiplier \(\gamma_{SH}\) has component coefficients that account for the change in conditions enumerated in Section 4.4.1 (General Shrinkage Behavior). ACI Committee 435 (1995) recommends, in detailed tabular form, the various component coefficients for the \(\gamma_{SH}\) multiplier. These are generally based on Branson’s (1977) studies. The tabulated values (ACI Committee 435, 1995; Branson, 1977; Meyers and Thomas, 1983) are given in graphical form in Figure 4.16 for:

\[
\gamma_{SH} = K_{H}^{'}K_{d}K_{s}K_{F}K_{B}K_{AC}
\]  
(4.12c)

where values of these factors are given in Figure 4.13:

- \(\gamma_{SH} = 1\) for standard conditions.
- \(K_{H}^{'}\) = relative humidity-factor.
- \(K_{d}^{'}\) = minimum member thickness factor.
- \(K_{s}^{'}\) = slump factor.
- \(K_{F}^{'}\) = fine aggregate content factor.
- \(K_{B}^{'}\) = cement-content factor.
- \(K_{AC}^{'}\) = air-content factor.

### 4.4.4 Alternate Method for Shrinkage Prediction in Prestressed Concrete Elements

For standard conditions, the Precast/Prestressed Concrete Institute (PCI) stipulates an average value for nominal ultimate shrinkage strain \((\varepsilon_{SH})_{u} = 820 \times 10^{-6}\) in./in. (mm/mm). If \(\varepsilon_{SH}\) is the shrinkage strain after adjusting for relative humidity (RH, in percent) at a volume-to-surface ratio \((V/S, \text{ in inches})\), the shrinkage strain is:

\[
\varepsilon_{SH} = 8.2 \times 10^{-6}K_{SH} \left[ 1 - 0.06 \frac{V}{S} (100 - \text{RH}) \right]
\]  
(4.13)

where \(K_{SH} = 1.0\) for pretensioned members. Table 4.2 gives the values of \(K_{SH}\) for post-tensioned members. Adjustment of shrinkage losses for standard conditions as a function of time \(t\), in days, is made using Equations 4.10a and 4.10b for standard conditions and Equations 4.12a and 4.12b for nonstandard conditions.

<table>
<thead>
<tr>
<th>Time from End of Moist Curing to Application of Prestress (Days)</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_{SH})</td>
<td>0.92</td>
<td>0.85</td>
<td>0.80</td>
<td>0.70</td>
<td>0.73</td>
<td>0.64</td>
<td>0.58</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Source: Precast/Prestressed Concrete Institute (PCI), Chicago, IL.
4.4.5 CEB-FIP Model Code for Shrinkage

The CEB-FIP 90 model proposed by the Euro-International Concrete Committee and the International Federation for Prestressing is based on Muller and Hillsdorf (1990) and is only applicable for concrete with 28-day compressive strength in the range of 20 to 90 MPa. The input parameters of this model differ from the ACI model in terms of compressive strength and type of curing method. The strain due to shrinkage at normal temperature may be calculated from:

\[ \varepsilon_{\text{SH(t,s)}} = \varepsilon_{\text{so}} \beta_s (t - t_s) \]  

(4.14)

where:
- \( \varepsilon_{\text{so}} \) = shrinkage coefficient.
- \( \beta_s \) = coefficient to describe shrinkage with time.
- \( t \) = age of concrete (days).
- \( t_s \) = age of concrete (days) at the beginning of shrinkage.

4.4.6 Effects of Water and Slump on Drying Shrinkage

Traditionally, it has been accepted that the higher the water/cement ratio (w/c) and, correspondingly, the higher the water/cementitious materials ratio (w/cm), the higher the shrinkage effects. This concept dates back to the early days of the 1930s and is exemplified by the relationship shown in Figure 4.16, which relates shrinkage to aggregate content and, significantly, to the water/cement ratio. More recent work (Suprenant and Malisch, 2000) has produced results that bring into question the degree of influence on the drying shrinkage value by the increase in slump resulting from an increase in water content in the mix. The indicated work, monitoring the increase in drying shrinkage over a 1-year period, demonstrated that only a 5% increase in shrinkage resulted for each 2-inch increase in slump: “As a general rule of thumb, each added gallon of water per cubic yard increases slump by 1 inch” (Suprenant and Malisch, 2000). Figure 4.15 graphically illustrates that only a slight increase in average drying shrinkage results from appreciable increase in water content. The authors proposed in their investigation that, with proper inspection, jobsite water additions seldom exceed 2 gallons of water per cubic yard of concrete, or enough to increase the slump 2 inches. On this basis, the 2 gallons of water, or the extra 2 inches of slump, “might increase drying shrinkage by 4% to 5%,” which is not significant over a 1-year period after placement of the concrete.

4.5 Strength and Elastic Properties of Concrete vs. Time

4.5.1 Cylinder Compressive Strength \( (f'_{c}) \)

Cylinder compressive strength increases with time as the cement hydration reaction progresses in the presence of water. As a function of time, the developing compressive strength is:

\[ (f'_{c}) = \frac{1}{\alpha/\beta + t} (f'_{c})_u \]  

(4.15)

where:
- \( \alpha/\beta \) = age of concrete, in days, at which one half of the ultimate (in time) compressive strength of concrete \( (f'_{c})_u \) is reached.
- \( t \) = age of concrete in days.

The range of \( \alpha \) and \( \beta \) for normal weight, sand lightweight, and all lightweight concrete is \( \alpha = 0.05 \) to 9.25, and \( \beta = 0.67 \) to 0.98. These constants are a function of the type of cement and the type of curing applied. Typical values for \( \alpha/\beta \) and the time strength ratios are given in Table 4.3.
4.5.2 Modulus of Rupture \((f_r)\) and Tensile Strength \((f_t')\)

The modulus of rupture \((f_r)\) can be expressed as:

\[
f_r = g_r \sqrt{w(f'_c)}
\]  
(4.16)

where \(g_r\) has a range of 0.6 to 1.00, with an average value of 0.65 (in SI units, this range is 0.012 to 0.021, with an average of 0.0135 MPa for \(f_r\)); \(w\) is the unit weight of the concrete in pounds per cubic foot for \(f_r\) in psi or kg/m\(^3\) for \(f_r\) in megapascals. Hence, Equation 4.16 becomes:

\[
f_r, \text{ (psi)} = 0.65 \sqrt{w f'_c}
\]  
(4.17a)

and

\[
f_r, \text{ (MPa)} = 0.013 \sqrt{w f'_c}
\]  
(4.17b)

Equation 4.17 is applicable for concrete strengths up to 12,000 psi (83 MPa). For normal weight concrete, \(w = 145 \text{ lb/ft}^3\ (2320 \text{ kg/m}^3)\), and Equation 4.17 becomes:

\[
f_r, \text{ (psi)} = 7.5 \sqrt{f'_c}
\]  
(4.18a)

and

\[
f_r, \text{ (MPa)} = 0.60 \sqrt{f'_c}
\]  
(4.18b)

ACI Committee 363 (1992) on high-strength concrete (Nawy, 1996, 2001) recommends higher values for the modulus of rupture for normal weight concrete:

\[
f_r, \text{ (psi)} = 11.7 \sqrt{f'_c}
\]  
(4.19a)

and

\[
f_r, \text{ (MPa)} = 0.94 \sqrt{f'_c}
\]  
(4.19b)

The tensile splitting strength \(f_t'\) as recommended in ACI Committee 363 (1992) and ACI Committee 435 (1995) for normal weight concrete of a compressive-strength range up to 12,000 psi (83 MPa) is:

\[
f_t', \text{ (psi)} = 7.4 \sqrt{f'_c}
\]  
(4.20a)

and

\[
f_t', \text{ (MPa)} = 0.59 \sqrt{f'_c}
\]  
(4.20b)
4.5.3 Modulus of Elasticity ($E_c$)

The modulus of elasticity of concrete is strongly influenced by the concrete materials and mix proportions used. An increase in compressive strength is accompanied by an increase in the modulus, as the slope of the ascending branch of the stress–strain diagram becomes steeper. For concretes with densities in the range of 90 to 155 lb/ft$^3$ (1440 to 2320 kg/m$^3$), based on the secant modulus at 0.45$f_c'$ intercept and compressive strength up to 6000 psi (42 MPa):

$$E_c (\text{psi}) = 33w^{1.5}\sqrt{f_c'}$$  \hspace{1cm} (4.21a)

and

$$E_c (\text{MPa}) = 0.0143w^{1.5}\sqrt{f_c'}$$  \hspace{1cm} (4.21b)

As the strength of the concrete increases beyond 6000 psi, the measured value of $E_c$ increases at a slower rate such that the value expressed in Equation 4.21 underestimates the actual value of the modulus. The value of the modulus for a compressive strength range of 6000 to 12,000 psi (42 to 83 MPa) (Nilson, 1985) can be predicted by:

$$E_c (\text{psi}) = \left\{40,000\sqrt{f_c'} + 1.0\times10^8\right\}\left(\frac{w_c}{145}\right)^{1.5}$$  \hspace{1cm} (4.22a)

and

$$E_c (\text{MPa}) = \left\{3.32\sqrt{f_c'} + 6895\right\}\left(\frac{w_c}{2320}\right)^{1.5}$$  \hspace{1cm} (4.22b)

Figure 4.17 gives the best fit for $E_c$ vs. $f_c'$ for high-strength concretes. Deviations from the predicted values are highly sensitive to properties of the coarse aggregate such as size, porosity, and hardness. When very high-strength concretes (20,000 psi or 140 MPa, or higher) are used in major structures or when deformation is critical, $E_c$ should be determined from actual field cylinder test values and the 0.45$f_c'$ intercept in the resulting stress–strain diagram. Long-term effects on the modulus of elasticity can be viewed in terms of the gain in the compressive strength ($f_c'$), such that:

$$E_{ct} = E_c\sqrt{\left[f_c'/f_c\right]}$$  \hspace{1cm} (4.23)

where ($f_c'$) is equal to compressive strength at later ages and ($f_c$) is equal to the 28-day compressive strength.

4.6 Serviceability Long-Term Considerations

In concrete structural members, serviceability is evaluated by cracking and deflection behavior. Creep and shrinkage effects on cracking and deflection are well established. Both deflections and crack widths increase with time. As a section cracks, its gross moment of inertia is reduced, resulting in reduced stiffness and larger deformations and deflections. The crack width ($w$) and the cracking moment ($M_{cr}$), are the principal parameters, together with the contribution of the reinforcement (compressive reinforcement in the case of deflection), that determine the long-term behavior of structural elements and systems.

4.6.1 Cracking Moment ($M_{cr}$) and Effective Moment of Inertia ($I_e$)

4.6.1.1 Reinforced Concrete Beams

Tension cracks develop when externally imposed loads cause bending moments in excess of the cracking moment, $M_{cr}$. As a result, tensile stresses in the concrete at the tensile extreme fibers exceed the modulus...
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of rupture ($f_r$) of the concrete. The cracking moment for an uncracked section can be computed from the basic flexural formula:

$$M_{cr} = \frac{f_r I_g}{y_t}$$ (4.24a)

or

$$M_{cr} = \frac{f_r}{S_t}$$ (4.24b)

where:

- $I_g$ = gross moment of inertia.
- $y_t$ = distance form the neutral axis to the extreme tension fibers.
- $f_r$ = modulus of rupture.
- $S_t$ = section modulus to the extreme tension fibers.

Cracks develop at several sections along a member length. At those sections where the modulus of rupture ($f_r$) is exceeded, cracks develop and the moment of inertia is reduced to a cracked moment ($I_{cr}$). At other sections where cracks do not develop, $I_g$ is used for evaluating the stiffness of the sections.

Branson’s work (1977), the basis of the ACI 318 Code, proposed using the effective moment of inertia ($I_e$) for cracked sections:

$$I_e = \left( \frac{M_e}{M_g} \right)^3 I_g + \left[ 1 - \left( \frac{M_e}{M_g} \right)^3 \right] I_{cr} \leq I_g$$ (4.25)
where:

- $M_{cr}$ = cracking moment.
- $M_a$ = maximum moment at the stage at which deflections are being considered.
- $I_g$ = gross moment of inertia of the section.
- $I_{cr}$ = moment of inertia of the cracked transformed section.

The two moments, $I_g$ and $I_{cr}$, are based on the assumption of bilinear load-deflection behavior, as seen in Figure 4.18 (Nawy, 2008). The cracked moment of inertia ($I_{cr}$) is:

$$I_{cr} = n A_d d \left( 1 - 1.6 \sqrt{n \rho} \right)$$  \hspace{1cm} (4.26)

where:

- $n$ = modular ratio $= E/E_c$.
- $\rho$ = $A_d/bd$.
- $d$ = effective depth.

Equation 4.25 can be rewritten as follows:

$$I_e = I_{cr} + \left( \frac{M_{cr}}{M_a} \right)^3 (I_g - I_{cr}) \leq I_g$$  \hspace{1cm} (4.27)

For continuous beams, ACI 318 allows $I_e$ to be taken as the average value obtained from Equation 4.25 or Equation 4.26 for the critical positive and negative moment sections. For prismatic sections, $I_e$ may be taken as the value obtained at midspan for continuous spans. If the designer chooses to average the effective moments of inertia ($I_e$), the following expression can be used:

$$I_e = 0.5 I_{cr,1} + 0.25 (I_{cr,1} + I_{cr,2})$$  \hspace{1cm} (4.28)

where $m$, 1, and 2 refer to midspan and the two beam ends, respectively. Improved results for continuous prismatic members can be obtained by using a weighted average (ACI Committee 435, 1995) for beams that are continuous on both ends:

$$I_e = 0.70 I_{cr,1} + 0.15 (I_{cr,1} + I_{cr,2})$$  \hspace{1cm} (4.29a)
4.6.1.2 Prestressed Concrete Beams

The effective moment of inertia ($I_e$) in Equation 4.25 and Equation 4.27 is based on different moment levels for $M_{cr}$ and $M_a$ in the case of prestressed concrete beams because of the initial compressive stress imposed by the prestressing force. The $M_{cr}/M_a$ value is defined by:

$$\left(\frac{M_{cr}}{M_a}\right) = \left(1 - \frac{f_{TL} - f_L}{f_L}\right)$$

(4.30)

where:

$$f_L = 7.5\lambda\sqrt{f_{cr}}$$

Here, $\lambda = 1.0$ for normal weight concrete; $\lambda = 0.85$ for sand lightweight concrete; $\lambda = 0.75$ for all lightweight concrete, and:

- $M_{cr}$ = moment due to that portion of the unfactored live-load moment that causes cracking.
- $M_a$ = maximum unfactored live-load moment.
- $f_{TL}$ = total calculated stress in the member.
- $f_L$ = calculated stress owing to live load.

In prestressed beams that are partially prestressed by the addition of mild steel reinforcement:

$$I_e = \left(n_y A_y d_y^2 + n_x A_x d_x^3\right) \left[1 - 1.6\sqrt{n_y \rho_y + n_x \rho_x}\right]$$

(4.31)

4.6.1.3 Effect of Compression Reinforcement

Compression reinforcement in reinforced flexural members and nontensioned reinforcement such as mild steel in prestressed flexural members tend to offset the movement of the neutral axis caused by creep (ACI Committee 209, 1992). A reverse movement toward the tensile fibers can thus result. A multiplier ($\gamma$) has to be used to account for increase in deflection, as required in the ACI 318 Building Code:

$$\gamma = \frac{\xi}{1 + 50\rho'}$$

(4.32)

where:

- $\xi$ = time-dependent factor for the long-term increase in deflection obtained from Figure 4.19 (ACI Committee 435, 1995).
- $\rho' = A'_{c}/bd$.
- $A'_{c}$ = area of compression reinforcement (in square inches).

Nilson (1985) suggested that two modifying factors should be applied to Equation 4.32: the material modifier ($\mu_m$) to be applied to $\xi$ and the section modifier ($\mu_s$) to be applied to $\rho'$. Both $\mu_m$ and $\mu_s$ have a value of 1 or less. Combining the two multipliers, without significant loss in accuracy, Equation 4.32 becomes:

$$\gamma = \frac{\mu_s \xi}{1 + 50\mu_s \rho'}$$

(4.33)
with the range of $\mu$ values as follows within the 6000- to 9500-psi (42- to 66-MPa) compressive strength tests conducted:

$$\mu \geq 0.7$$

or, megapascals for $f'_c$:

$$\mu \leq (1.3 - 0.00005f'_c) \leq 1.0$$

Further evaluations are needed for cases where the concrete strength is higher than 12,000 psi (83 MPa).

### 4.6.2 Flexural Crack Width Development

External load results in direct and bending stresses, which cause flexural, bond, and diagonal tension cracks. Immediately after the tensile stress in the concrete exceeds its tensile strength at a particular location, any internal microcracks that might have formed begin to propagate into macrocracks. These cracks develop into macrocracks that propagate to the external fiber zones of the element. Immediately after full development of the first crack in a reinforced concrete element, the stress in the concrete at the cracking zone is reduced to zero and is assumed by the reinforcement (Nawy, 2001, 2008) The distribution of ultimate bond stress ($\mu$), longitudinal stress in the concrete ($f_l$), and longitudinal tensile stress ($f_s$) in the reinforcement can be schematically represented as shown in Figure 4.20. Crack width is primarily a function of the deformation of reinforcement between the two adjacent cracks, 1 and 2 (Figure 4.20), if the small concrete tensile strain along the crack interval $a_i$ is neglected. The crack width is thus a function of crack spacing up to the load level at which no more cracks develop, which leads to stabilization of the crack spacing, as shown in Figure 4.21. The major parameters affecting the development and characteristics of cracks are percentage of reinforcement, bond characteristics and size of bar, concrete cover, and the concrete area in tension. On this basis, one can propose the following mathematical model:

$$w = \alpha a_i^\beta \varepsilon_i^\gamma$$

where $w$ is maximum crack width and $\alpha$, $\beta$, and $\gamma$ are nonlinearity constants. Crack spacing ($a_i$) is a function of factors to be subsequently discussed and is inversely proportional to bond strength and active steel ratio (steel percentage in terms of the concrete area in tension). Here, $\varepsilon_i$ is the strain in the reinforcement induced by the external load.
FIGURE 4.20 Schematic stress distribution between two flexural cracks. (From Nawy, E.G. and Blair, K.W., in Cracking, Deflection, and Ultimate Load of Concrete Slab Systems, ACI SP-30, Nawy, E.G., Ed., American Concrete Institute, Farmington Hills, MI, 1971, pp. 1–41.)

FIGURE 4.21 Schematic variation of crack width with crack spacing. (From Nawy, E.G. and Blair, K.W., in Cracking, Deflection, and Ultimate Load of Concrete Slab Systems, ACI SP-30, Nawy, E.G., Ed., American Concrete Institute, Farmington Hills, MI, 1971, pp. 1–41.)
The basic mathematical model (Equation 4.35), with the appropriate experimental values of the constants \( \alpha, \beta, \) and \( \gamma \), can be derived for a particular type of structural member. Such a member can be a one-dimensional element such as a beam, a two-dimensional structure such as a two-way slab, or a three-dimensional member such as a shell or circular tank wall. Hence, it is expected that different forms or expressions apply for evaluation of macrocracking behavior of different structural elements consistent with their fundamental structural behavior (ACI Committee 224, 2001; ACI Committee 318, 2008; CEB-FIP, 1990; Gergely and Lutz, 1968; Nawy, 1972a,b, 1994; Nawy and Blair, 1971).

4.6.2.1 Reinforced Concrete Beams and One-Way Slabs

The requirements for crack control in beams and thick one-way slabs—10 in. (250 mm) or thicker—in the ACI Building Code (ACI Committee 318, 2008) are based on the statistical analysis of maximum crack width data from a number of sources. On the basis of the analysis, the following general conclusions were reached (Gergely and Lutz, 1968; Nawy, 1972a,b, 1994; Nawy and Blair, 1971):

1. The steel stress is the most important variable.
2. The thickness of the concrete cover is an important variable.
3. The area of concrete surrounding each reinforcing bar is an important geometric variable.
4. Bar diameter is not a major variable.
5. The bottom crack width is influenced by the amount of strain gradient from the level of the steel to the tension face of the beam.

The simplified expression relating crack width to steel stress is (Gergely and Lutz, 1968):

\[
\text{Equation 4.36a: } \quad w_{\text{max}} \text{ (in.)} = 0.076\beta \sqrt{\frac{f_s}{A}} \times 10^{-3}
\]

where:
- \( f_s \) = reinforcing steel stress (ksi).
- \( A \) = area of concrete symmetrical with reinforcing steel divided by number of bars (in.²).
- \( d_c \) = thickness of concrete cover measured from extreme tension fiber to center of bar or wire closest thereto (in.).
- \( \beta = \frac{h_2}{h_1} \), where \( h_1 \) is the distance from the neutral axis to the reinforcing steel (in.), and \( h_2 \) is the distance from the neutral axis to the extreme concrete tensile surface.

When the strain \( (\varepsilon_s) \) in the steel reinforcement is used instead of stress \( (f_s) \), Equation 4.36a becomes:

\[
\text{Equation 4.36b: } \quad w = 2.2\beta \varepsilon_s \sqrt{d_c A}
\]

Equation 4.36b is valid in any system of measurement.

The cracking behavior in thick one-way slabs is similar to that in shallow beams. For one-way slabs that have a clear concrete cover in the range of 1 in. (25.4 mm), Equation 4.38 can be adequately applied if \( \beta = 1.25 \) to 1.35 is used.

The ACI 318 Building Code currently uses reinforcement spacing as the criteria for control of the crack width. It recommends for beams and one-way slabs the following expression:

\[
\text{Equation 4.37: } \quad s \text{ (in.)} = 15 \left( \frac{40,000}{f_s} \right) - 2.5c
\]

but not greater than \( 12(40,000/f_s) \), where:
- \( s \) = spacing (in.).
- \( f_s \) = calculated reinforcement stress at service level or alternatively taken as \( 2/3 \) the yield strength \( f_y \) (psi).
- \( c \) = clear cover to the reinforcement (in.).
The SI expression in Equation 4.37 is:

\[ s \text{ (mm)} = 380 \left( \frac{280}{f_{t}'} \right) - 2.5s_c \]  

but not to exceed \(300(280/f_{t}')\), where \(f_{t}'\) in usual cases is taken as 252 MPa.

It should be noted that the ACI expression is applicable to reinforced concrete beams and one-way slabs in normal environmental conditions. In aggressive environments, such as liquid-retaining sanitary structures, Equations 4.36a and 4.36b, with the appropriate limitation of tolerable crack widths listed in Table 4.4, are more appropriate in such cases.

### 4.6.2.2 Prestressed Concrete Beams

#### 4.6.2.2.1 Crack Spacing

Primary cracks form in the region of maximum bending moment when the external load reaches the cracking load. Sometimes, in post-tensioned parking garage elements, cracks form in the draped region before forming at the maximum moment region. They can also form at debonded tendon locations. As loading is increased, additional cracks will form, and the number of cracks will be stabilized when the stress in the concrete no longer exceeds its tensile strength at further locations, regardless of load increase.

This condition is important, as it essentially produces the absolute minimum crack spacing that can occur at high steel stresses, and is termed the stabilized minimum crack spacing. The maximum possible crack spacing under this stabilized condition is twice the minimum and is termed the stabilized maximum crack spacing. Hence, the stabilized mean crack spacing \((a_{mc})\) is deduced as the mean value of the two extremes. The total tensile force \((T)\) transferred from the steel to the concrete over the stabilized mean crack spacing (Nawy, 1990, 1994, 1996) can be defined as:

\[ T = \gamma \mu \sum \rho \]  

where:

- \(\gamma\) = a factor reflecting the distribution of bond stress.
- \(\mu\) = maximum bond stress that is a function of \(\sqrt{f_t'}\).
- \(\sum \rho\) = sum of reinforcing elements' circumferences.

The resistance \((R)\) of the concrete area in tension \((A_b)\) can be defined as:

\[ R = A_b f_t' \]  

where \(f_t'\) is the tensile splitting strength of the concrete.
By equating Equations 4.39a and 4.39b, the following expression for \( a_{cs} \) is obtained, where \( c \) is a constant to be developed from the tests:

\[
a_{cs} = c \sum A_i f'_{iy} \quad (4.40a)
\]

The concrete stretched area (namely, the concrete area \( A_i \) that is under tension for both the evenly distributed and unevenly distributed reinforcing elements) is illustrated in Figure 4.23. With a mean value of:

\[
f'_{iy} / \sqrt{f'_{iy}} = 7.95
\]

the mean stabilized crack spacing becomes:

\[
a_{cs} = 1.20 \sum A_i \quad (4.40b)
\]

4.6.2.2 Crack Width

If \( \Delta f_i \) is the net stress in the prestressed tendon or the magnitude of the tensile stress in normal steel at any crack-width load level in which the decompression load (decompression here means \( f'_{iy} = 0 \) at the
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The level of the reinforcing steel (Nawy, 1990, 1996), then for the prestressed tendon:

\[ \Delta f_s = f_{ns} - f_d \] (4.41)

where:
- \( f_{ns} \) = stress in the prestressing steel at any load beyond the decompression load.
- \( f_d \) = stress in the prestressing steel corresponding to the decompression load.

Unit strain \( \varepsilon = \Delta f_s / E_s \). It is logical to disregard as insignificant unit strains in the concrete caused by temperature, shrinkage, and elastic shortening effects. The maximum crack width as defined in Equation 4.35 can therefore be taken as:

\[ w_{max} = k_{ns} \varepsilon \] (4.42a)

or

\[ w_{max} = k' a_{us} (\Delta f_s)^u \] (4.42b)

where \( k' \) is a constant in terms of constant \( k \).

**4.6.2.2.3 Expression for Pretensioned Beams**

Equation 4.42b is rewritten in terms of \( \Delta f_s \) to give the maximum crack width at the reinforcement level as follows:

\[ w_{max} \text{ (in.)} = 5.85 \times 10^{-5} A_t (\Delta f_s) \sum a \] (4.43a)

where \( A_t \) = square inches, \( \Sigma a = \) inches, and \( \Delta f_s = \) kips/in.2, and

\[ w_{max} \text{ (mm)} = 8.48 \times 10^{-5} A_t (\Delta f_s) \sum a \] (4.43b)

where \( A_t = \) cm2, \( \Sigma a = \) cm, and \( \Delta f_s = \) MPa.

**FIGURE 4.23** Effective concrete area in tension: (a) even reinforcement distribution; (b) non-even reinforcement distribution. (From Nawy, E.G., in *Cracking in Prestressed Concrete Structures*, ACI SP-113, American Concrete Institute, Farmington Hills, MI, 1990, pp. 1–42.)
The maximum crack width (inches) at the tensile face of the concrete is:

\[ w_{\text{max}} = 5.85 \times 10^{-5} R_i \sum_o A_o (\Delta f_o) \]  

(4.43c)

where \( R_i \) is the distance ratio equal to \( h_2/h_1 \), where \( h_2 \) is the distance from the neutral axis to the extreme tension fibers and \( h_1 \) is the distance from the neutral axis to the reinforcement centroid. A plot of the pretensioned beams test data and the best fit expression for Equation 4.43a is given in Figure 4.24 with a 40% spread, which is reasonable in view of the randomness of crack development.

### 4.6.2.2.4 Expressions for Post-Tensioned Beams

The expression developed for the crack width in post-tensioned bonded beams that contain mild steel reinforcement is:

\[ w_{\text{max}} \text{ (in.)} = 6.51 \times 10^{-5} \sum_o A_o (\Delta f_o) \]  

(4.44a)

and

\[ w_{\text{max}} \text{ (mm)} = 9.44 \times 10^{-5} \sum_o A_o (\Delta f_o) \]  

(4.44b)

for the crack width at the reinforcement level closest to the tensile face.

At the tensile face, the crack width for the post-tensioned beams becomes:

\[ w_{\text{max}} \text{ (in.)} = 6.51 \times 10^{-5} R_i \sum_o A_o (\Delta f_o) \]  

(4.44c)

For nonbonded beams, the factor 6.51 in Equations 4.44a and 4.44c becomes 6.83. A plot of the data and the best-fit expression for Equation 4.44a is given in Figure 4.25. A typical plot of the effect of the various steel percentages on the crack spacing at various stress levels (\( \Delta f_i \)) is given in Figure 4.26. It can be seen from this plot that crack spacing stabilizes at a net stress level range of 30 to 36 kips/in.\(^2\) (207 to 248 MPa).
4.6.2.2.5 Cracking of High-Strength Prestressed Beams

Analysis of continuing subsequent work by the first author on the cracking behavior of pretensioned and nonbonded post-tensioned beams having cylinder compressive strengths in the range of 10,200 to 14,200 psi (70.3 to 97.9 MPa) have resulted in the following expression for crack width at the reinforcement level of pretensioned members:

\[
W_{\text{max}} = 6.51 \times 10^{-5} \frac{A_r}{\Sigma o} \Delta f_i
\]  \hspace{1cm} (4.45a)

and

\[
w_{\text{max}} \text{ (in.)} = 2.75 \times 10^{-3} \sum o \frac{A_r}{\Sigma o} (\Delta f_i)
\]  \hspace{1cm} (4.45b)


4.6.2.2.5 Cracking of High-Strength Prestressed Beams

Analysis of continuing subsequent work by the first author on the cracking behavior of pretensioned and nonbonded post-tensioned beams having cylinder compressive strengths in the range of 10,200 to 14,200 psi (70.3 to 97.9 MPa) have resulted in the following expression for crack width at the reinforcement level of pretensioned members:

\[
w_{\text{max}} \text{ (in.)} = 2.75 \times 10^{-3} \sum o \frac{A_r}{\Sigma o} (\Delta f_i)
\]  \hspace{1cm} (4.45a)

and

\[
w_{\text{max}} \text{ (mm)} = 4.0 \times 10^{-3} \sum o \frac{A_r}{\Sigma o} (\Delta f_i)
\]  \hspace{1cm} (4.45b)
The factor 2.75 is an average of values from the following statistical expression (Nawy, 1994) for a reduction multiplier \( \lambda_r \) of \( w_{\text{max}} \) in Equation 4.43 such that:

\[
\lambda_r = \frac{2}{0.75 + 0.06 \sqrt{f'_t}} \sqrt{f'_c} 
\]  
\[ (4.45c) \]

This reduced crack width due to use of high-strength concrete is expected in view of the increased bond interaction between the concrete and the reinforcement.

### 4.6.2.2.6 Other Work on Cracking in Prestressed Concrete

After analyzing results from various investigators (Harajli and Naaman, 1989; Naaman and Siriakson, 1979), Naaman produced the following modified expression for partially prestressed pretensioned members:

\[
w_{\text{max}} = \left( 42 + 5.52 \times 10^{-5} \right) \sum_{\rho} \frac{A_t}{A_c} (\Delta f'_o) \times 10^{-5} 
\]  
\[ (4.46) \]

This expression is very close to that in Equation 4.43 by Nawy. If plotted against results of the various researchers’ work, it gives a best fit as shown in Figure 4.27.

### 4.6.2.3 Two-Way Supported Slabs and Plates

Flexural crack control is essential in structural floors, most of which are under two-way action. Cracks at service-load and overload conditions can be serious in floors such as those in office buildings, schools, parking garages, and industrial buildings and in other floors where the design service load and overload levels exceed loads in normal-size apartment building panels. Such cracks can only lead to detrimental effects on the integrity of the total structure, particularly in adverse environmental conditions.

#### 4.6.2.3.1 Flexural Cracking Mechanism and Fracture Hypothesis

Flexural cracking behavior in concrete structural floors under two-way action is significantly different from that in one-way members. Crack control equations for beams underestimate crack widths developed in two-way slabs and plates and do not tell the designer how to space reinforcement. Cracking in two-way slabs and plates is primarily controlled by the steel stress level and the spacing of reinforcement in two perpendicular directions. In addition, the clear concrete cover in two-way slabs and plates is nearly
constant (3/4 in., or 20 mm, for interior exposure), whereas it is a major variable in the crack control equations for beams. Results from extensive tests on slabs and plates (Nawy, 1972a, 1994; Nawy and Blair, 1971) demonstrate this difference in behavior in a fracture hypothesis on crack development and propagation in two-way plate action. Nawy’s work also conclusively demonstrated that surface deformations of individual reinforcing elements have little effect on arresting the generation of cracks or controlling crack type or width in a two-way-action slab or plate. One may also conclude that the scale effect on two-way-action cracking behavior is insignificant, as the cracking grid is a reflection of the reinforcement grid if the preferred orthogonal narrow cracking widths develop. To control cracking in two-way-action floors, then, the major parameter to be considered is the reinforcement spacing in the two perpendicular directions. Concrete cover has only a minor effect, as the cover is usually small, with a constant value of 0.75 in. (20 mm). Maximum spacing of the reinforcement in both orthogonal directions should not exceed 12 in. (30 cm) in any structural floor.

4.6.2.3.2 Crack Control Equation

The basic Equation 4.35 for relating crack width to strain in the reinforcement is:

\[ w = \alpha a \beta \epsilon \gamma \]

The effect of the tensile strain in the concrete between the cracks is neglected as insignificant. The parameter \( a \) is the crack spacing, \( \epsilon \) is the unit strain in the reinforcement, and \( \alpha, \beta, \) and \( \gamma \) are constants evaluated by tests. The mathematical model in Equation 4.35 and statistical analysis of the data of 90 slabs tested to failure give the following equation (ACI Committee 224, 2001; Nawy and Blair, 1971) for serviceability requirements for crack control:

\[ w \text{ (in.)} = K \beta f_s \sqrt{G_I} \quad (4.47) \]

Using SI units, the expression becomes:

\[ w_{\text{max}} \text{ (mm)} = 0.145K f_s \sqrt{G_I} \quad (4.48) \]

where \( f_s \) is in megapascals, and all the terms for the grid index \( (G_I) \) in Equation 4.49 are in millimeters. \( G_I = d_1 s_2 / \rho t \) is the grid index that defines the reinforcement distribution in two-way action slabs and plates. It can be transformed in Equation 4.47 to:

\[ G_I = \frac{s_1 s_2 d_1}{\rho t} \frac{8}{\pi} \quad (4.49) \]

where:

\( K \) = fracture coefficient, having a value of \( K = 2.8 \times 10^{-5} \) for uniformly loaded restrained two-way action square slabs and plates. For concentrated loads or reactions, or when the ratio of short to long span is less than 0.75 but larger than 0.5, a value of \( K = 2.1 \times 10^{-5} \) is applicable. For a span-aspect ratio of 0.5, \( K = 1.6 \times 10^{-5} \). Units of coefficient \( K \) are in square inch per pound. (See also Table 4.5.)

\( \beta \) = ratio of the distance from the neutral axis to the tensile face of the slab to the distance from the neutral axis to the centroid of the reinforcement grid (to simplify the calculations use, \( \beta = 1.25 \), although it varies between 1.20 and 1.35).

\( f_s \) = actual average service load reinforcement stress level, or 40% of the design yield strength (ksi).

\( d_{01} \) = diameter of the reinforcement in direction 1 closest to the concrete outer fibers (in.).

\( s_1 \) = spacing of the reinforcement in perpendicular direction 1 (in.), closest to the tensile face.

\( s_2 \) = spacing of the reinforcement in perpendicular direction 2 (in.).

\( l \) = direction of the reinforcement closest to the outer concrete fibers; this is the direction for which the crack control check is to be made.

\( \rho_{t1} \) = active steel ratio in direction 1:
where \( c_1 \) is the clear concrete cover measured from the tensile face of the concrete to the nearest edge of the reinforcing bar in direction 1, and \( w \) is the crack width (in.) at the face of the concrete caused by flexural load. Subscripts 1 and 2 pertain to the directions of reinforcement. Detailed values of the fracture coefficients for various boundary conditions are given in Table 4.5. A graphical solution for Equation 4.47 is given in Figure 4.28 for \( f_y = 60 \) ksi (414 MPa), \( f_s = 40\% \), and \( f_y = 40 \) ksi (165.5 MPa) for rapid determination of the reinforcement size and spacing needed for crack control.

Because cracking in two-way slabs and plates is primarily controlled by the grid intersections of the reinforcement, concrete strength is not of major consequence; hence, the value of crack width in two-way action predicted by Equation 4.47 should not be significantly affected if higher strength concretes are used in excess of 6000 psi (41.4 MPa). It has to be pointed out that in two-way normal-slab floors, the use of much higher strengths is not justified in economical terms.

### 4.6.2.3.3 Tolerable Crack Widths in Concrete Structures

The maximum crack width that a structural element should be permitted to develop depends on the particular function of the element and the reasonable guide to the tolerable average crack widths in concrete structures under various normally encountered environmental conditions. Its values are in close agreement with the Comité Eurointernational du Beton recommendations (CEB-FIP, 1990) foremost exposure conditions. The crack-control equation and guidelines presented are important not only for the control of corrosion in reinforcement but also for deflection control. The reduction of the stiffness \((EI)\) of the two-way slabs or plates due to orthogonal cracking when the tolerable crack widths in Table 4.6 are exceeded can lead to both short- and long-term excessive deflection. Deflection values that are several times greater than those anticipated in the design, including deflection due to construction loading, can be reasonably controlled through camber and control of the flexural crack width in the slab or plate. Proper selection of reinforcement spacing \( s_1 \) and \( s_2 \) in the perpendicular directions, as discussed in this section, that does not exceed 12 in. (30 cm) center-to-center can maintain the good serviceability performance of a slab system under normal and reasonable overload conditions.

### 4.6.2.3.4 Long-Term Effects on Cracking

In most cases, the magnitude of crack widths increases in long-term exposure and long-term loading. Increase in crack width can vary considerably in cases of cyclic loading, such as in bridges; however, the crack width increases at decreasing rate with time. In most cases, a doubling of crack width after several years under sustained loading is not unusual.
4.6.2.4 Cracking in Circular Prestressed Concrete Tanks

Circular prestressed tanks are cylindrical shell elements of very large diameter in relation to their height; hence, with respect to flexural cracking, it is possible to treat the wall of a tank in a manner similar to the treatment of two-way action plates. Vessey and Preston (1978) modified Nawy and Blair’s (1971) expressions for two-way action slabs and plates so the maximum crack width can be defined as:

\[ W = K \beta f_y \sqrt{G_i} \]

where:
- \( W \) is the crack width
- \( K \) is a constant
- \( \beta \) is a factor
- \( f_y \) is the yield strength of the reinforcement
- \( G_i \) is the grid index

\[ G_i = \frac{d_{s1} \beta_2 \rho t_1}{\rho t_1} \]

\( f_y = 60 \text{ KSI} \)
\( f_t = 24 \text{ KSI} (= 0.40 f_y) \)

\[ K = 4.2 \times 10^{-5} \quad 3.4 \times 10^{-5} \quad 3.0 \times 10^{-5} \quad 2.8 \times 10^{-5} \quad 2.6 \times 10^{-5} \quad 2.4 \times 10^{-5} \quad 2.0 \times 10^{-5} \quad 1.8 \times 10^{-5} \quad 1.6 \times 10^{-5} \]

\[ 0.10 \quad 0.004 \quad 0.20 \quad 0.008 \quad 0.30 \quad 0.012 \quad 0.40 \quad 0.016 \quad 0.50 \quad 0.020 \quad 0.60 \quad 0.024 \quad 0.70 \quad 0.028 \quad 0.80 \quad 0.032 \quad 0.90 \quad 0.036 \]

**TABLE 4.6 Minimum Shrinkage and Temperature Reinforcement**

<table>
<thead>
<tr>
<th>Length between Movement Joints (ft)</th>
<th>Minimum Temperature and Shrinkage Reinforcement Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 20</td>
<td>Grade 40: 0.0030</td>
</tr>
<tr>
<td>20 to less than 30</td>
<td>Grade 40: 0.0040</td>
</tr>
<tr>
<td>30 to less than 40</td>
<td>Grade 40: 0.0050</td>
</tr>
<tr>
<td>40 and greater</td>
<td>Grade 40: 0.0060</td>
</tr>
</tbody>
</table>

*a Maximum shrinkage and temperature reinforcement where movement joints are not provided.

Note: When using this table, the actual joint spacing shall be multiplied by 1.5 if no more than 50% of the reinforcement passes through the joint.


4.6.2.4 Cracking in Circular Prestressed Concrete Tanks

Circular prestressed tanks are cylindrical shell elements of very large diameter in relation to their height; hence, with respect to flexural cracking, it is possible to treat the wall of a tank in a manner similar to the treatment of two-way action plates. Vessey and Preston (1978) modified Nawy and Blair’s (1971) expressions for two-way action slabs and plates so the maximum crack width can be defined as:
Using SI units, the expression becomes:

\[
 w_{\text{max}} (\text{mm}) = 0.6 \times 10^{-6} E_{ps} \varepsilon_{ct} \sqrt{G_I}
\]  

(4.51)

where, in the SI expression, \( E_{ps} \) is in MPa, the dimensions of all the parameters of the grid index \( G_I \) are in millimeters, and

- \( \varepsilon_{ct} \) = tensile surface strain in the concrete = \( \lambda_t F_p / (E_{ps}) \).
- \( f_p \) = actual stress in the steel.
- \( f_{pi} \) = initial prestress before losses.
- \( \lambda_t = f_p / f_{pi} \).
- \( G_I = \text{grid index} = (s_1 s_2 d_c / d_{b1}) (8/\pi) \).
- \( d_{b1} = \text{diameter of steel in direction 1.} \)
- \( s_1 = \text{spacing of the reinforcement in direction 1 closest to the tensile face.} \)
- \( s_2 = \text{spacing of the reinforcement in direction 2.} \)
- \( d_c = \text{concrete cover to center of steel, inches.} \)

Note that \( w_{\text{max}} = 0.004 \text{ in.} \) (0.1 mm) should be the limit of crack width for liquid-retaining tanks.

### 4.7 Long-Term Shrinkage and Temperature Reinforcement Controlling Cracking Between Joints in Walls and Slabs of Liquid-Retaining Structures

Cracking resulting from these unavoidable short-term and long-term strain gradients has always been a problem for the designer to consider as a significant factor in any structural design; hence, the use of joints is inevitable. It would be rare to find a structural system with total stress relievers, where if enough reinforcement is provided in the direction of the induced forces to prevent the cracks from opening then no joint would be needed. It would be economically prohibitive, as the volume of reinforcement required to perform this task would be significant. Because the induced forces are highly indeterminate, engineering judgment has to be exercised in interpreting the imprecise guidelines in codes and the literature on the type and spacing of joints to control cracking, which often render conflicting solutions.

Table 4.6 and Figure 4.29 (ACI Committee 350, 2001) stipulate the reinforcement percentages of shrinkage and temperature reinforcement for effective control of cracks that is essential to maintain between joints in liquid-retaining structures. Whereas the maximum percentage in Table 4.6 is given as 0.5% for movement joints of 40 feet, the high rigidity of the foundation slab at the wall joint to the slab renders this percentage inadequate. The relative lower flexibility of the wall as compared to the stiff foundation slab at the lower wall segment makes the connection totally rigid with a low magnitude of rotation at the joint, as the wall is almost fixed at the base and free at the top. To prevent long-term vertical cracking concentration at the lower quarter of the wall height, it might even be prudent to use almost 1.0 to 1.25% horizontal reinforcement for the lowermost segment of the wall if the wall thickness is in the range of 30 in. Details regarding the design, spacing, and construction of joints in most types of structural systems for the control of long-term cracking are covered in Chapter 17 of this Handbook.
Shrinkage in early-age concrete is an important aspect in determining the long-term cracking performance of concrete. The methods of curing have a significant effect on the level of shrinkage strains that develop, so it is useful to observe the different performances expected from different methods of curing and also the effect of the constituent cementitious materials on the autogenous shrinkage performance of the system. Tests by Suksawang et al. (2005) have provided a measure of differences in strains (hence, stresses) at 0 to 7 days and up to 100 days of curing. Figure 4.30a (early-age performance) and Figure 4.30b (100-day performance) indicate that, after 28 days of curing, the dry-cured specimen exhibited steeper shrinkage levels than the others due to the fact that less internally adsorbed water was available to prevent a higher increase in shrinkage strains. Figure 4.31 shows the effect of the pozzolanic content in the mixture on the autogenous shrinkage, where concrete with fly ash outperformed the others. Figure 4.32 demonstrates the significant difference between the shrinkage performance of lightweight and normal weight aggregate concrete under identical dry curing conditions and having the same water/cementitious ratio (w/cm) of 0.29. Shrinkage in the lightweight aggregate concrete was almost stabilized with minimal increase in shrinkage strain after 36 hours at a shrinkage strain of 200 $\mu$ε, whereas in the case of normal weight concrete it was close to 700 $\mu$ε at the same time interval and continuing to increase at a fast rate. This behavior has been observed by many investigators and can be attributed to the fact that lightweight aggregate concrete has a higher inert moisture content; hence, as the cement hydrates, it is constantly supplied with adsorbed moisture, making the specimen, through internal curing, less susceptible to shrinkage.

**Acknowledgments**

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FIGURE 4.30 Effect of curing methods on concrete shrinkage: (a) early (autogenous) shrinkage; (b) long-term (drying) shrinkage.

FIGURE 4.31 Effect of pozzolanic material on autogenous shrinkage of concrete.
Long-Term Effects and Serviceability

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Uniform distribution of ingredients in high-performance concrete. (Photograph courtesy of Portland Cement Association, Skokie, IL.)